OUTLINE

1

Classical Wishart matrices:

1. Introduced by J. Wishart in 1928 in the context of multivariate statistics.
2. Definition: $W = XX^{\dagger}$, here X is a rectangular $(n \times p)$ matrix with no specific symmetries.
3. Constructed from n sets of uncorrelated, discretized in time, Gaussian random processes $x\ (t_j)$ as

$$X_{\ ,j} = x\ (t_j), \quad W_{\ ,} = \sum_{j=1}^{p} X_{\ ,j} X_{\ ,j}, \qquad , \quad = 1, \ldots, n$$

W is the most random correlation matrix.

Non-Hermitean Wishart random matrices (New!!)

In problems where time-series generating systems are two different systems and the interest is in studying the correlations between them (e.g. correlations between functioning of the left and right hemispheres of a brain) the correlation matrix takes the form $W = XY^{\dagger}$[2]. We call this the non-Hermitean Wishart random matrix;

1. $W_{\ ;\ } = \sum_{j=1}^{p} X_{\ ;j} Y_{\ ;j};\quad ;\quad = 1;\dots;n;\quad (p > n)$. The matrix is no longer Hermitean and its spectrum becomes complex valued.
2. What are the differences between well studied Ginibre and non-Hermitean Wishart?
3. In the following we consider non-Hermitean Wishart matrices with complex entries (= 2).
4. Strong relation to QCD inspired non-Hermitean Laguerre ensembles. [*J. C. Osborn, PRL,* **93** *222001 (2004); G. Akemann, Nucl.Phys. B* **730** *253(2005); G. Akemann, M.J. Phillips, H.-J. Sommers, arXiv:0911.1276*]

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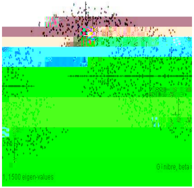
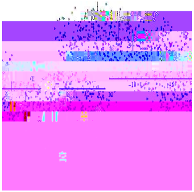
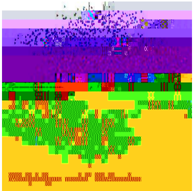
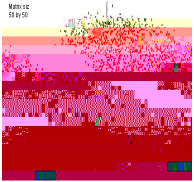
[2] J. Kwapien et al., PRE, 62, 5557 (2000)

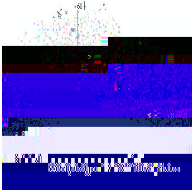
Results from computer experiments

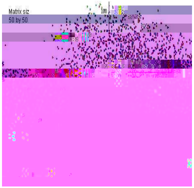


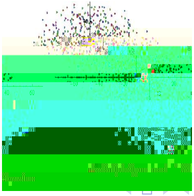


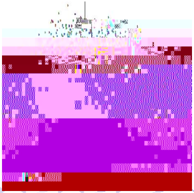












We solve by using successive size reduction of the matrix X_n, decompose X_n as:

1. $\mathbb{I} + X_n X_n^\dagger = \mathbb{I} + \begin{pmatrix} X_{n-1} & \\ u^\dagger & x_{nn} \end{pmatrix} \begin{pmatrix} X_{n-1}^\dagger & u \\ & x_{nn} \end{pmatrix}$

2. By decomposing $D\tilde{X}_n = (du \ \ du^\dagger) D\tilde{X}_{n-1}$, we derive,

97 T836 858Tf 6.9463Tf 2.87Td8 mnTJ 3.TJ841Tf 2.27 3.007

$$P_{n,p}(W)DW = C_U \int | \ _n(w)|^2 e^{i(w_j x_{jj}^\dagger + w_j x_{jj})}$$

The joint probability density function of all eigenvalues and the density of states

Solution by constructing a di erential equation and a scaling ansatz

1. De ne; $F_{p;n}(x) =$

Tails of the density (rare uctuations)

1. Breakdown of $\frac{1}{2|w|}$ – law at $|w| = |w_c|$:
   $2\int_0^{|w|_c} |w|\left(\frac{1}{2|w|}\right)d|w| = N; \quad \Rightarrow |w|_c = N$. Thus, something non-trivial should happen at about $|w| \sim N$. To understand it we have to treat the large N limit more carefully.
2. Idea is to utilize the Euler-Maclaurin formula to reduce the sum $F_{p;N} = \sum_{k=0}^{N-1} \frac{x^k}{k!(k+p)!}$ to an integral in the large N limit, and solve it using the saddle point approximation.
3. Case I: $|w| < N$

[redacted]

[redacted]

$p;N(|w|)w=$ $\sum_N$) i(01 11 1 1 (01 1) 2

Conclusions

- An exact treatment of non-Hermitean Wishart random matrices at $= 2$.
- Various large {N / large{ scaling regimes analyzed.
- Good agreement with numerical simulations observed.

Open problems

- The non-Hermitean Wishart at = 1 symmetry class (work in progress). In this case spectrum is complex valued as in the case = 2, but there is a nite probability of real eigenvalues (due to the accumulation of eigenvalues along the real axis).
- In applications where the number of discretizations (p) is less than number of channels (N) (for example, due to unavailability of data points), we are in the non-Hermetian anti-Wishart regime. This is a completely open area.
- **Experimental utilization of the present work remains open, although in such cases people have relied on Ginibre ensembles [see for example, J. Kwapien et al, arXive:physics/0605115]. The present work will improve our understanding of such applications.**