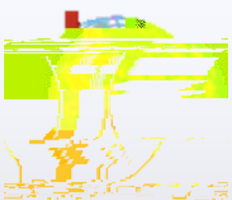
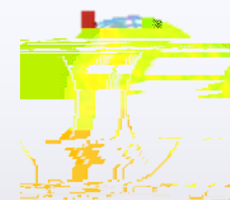
V BRUNEL Workshop on Random Matrix Theory

(Brunel University – West London)





by

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Coauthor: V. E. Kravtsov

Thanks: R. Balbinot, S. Fagnocchi & I. Carusotto
(also for the figures)

- Invariant Probability Distribution Function:

- Describe extended states (no localization)

  → Wigner statistics

- Gaussian Ensemble:

ritical Statistics

Spontaneous Breaking of Invariance?)

Weakly Confined Invariant Ensemble

$$V(E) \underset{|E|\to\infty}{\simeq} \frac{1}{\kappa} \ln^2 |E|, \qquad P(\mathbf{H}) \propto \mathrm{e}^{-\mathrm{tr}\, V(\mathbf{H})}$$

- Non-Trivial density eigenvalue distribution
- Unfolding to make density constant:

$$\rho(E) \equiv \mathrm{tr}\left\{\delta\left(E - \mathbf{H}\right)\right\}$$

$$E_x = \lambda\, \mathrm{e}^{\kappa |x|}\, \mathrm{sign}(x)$$

$$\langle \tilde{\rho}(x) \rangle = \langle \rho(E_x) \rangle \frac{\mathrm{d}E_x}{\mathrm{d}x} = 1$$

Weakly Confined Invariant Ensemble

[illegible] $|E|\to\infty$ [illegible]

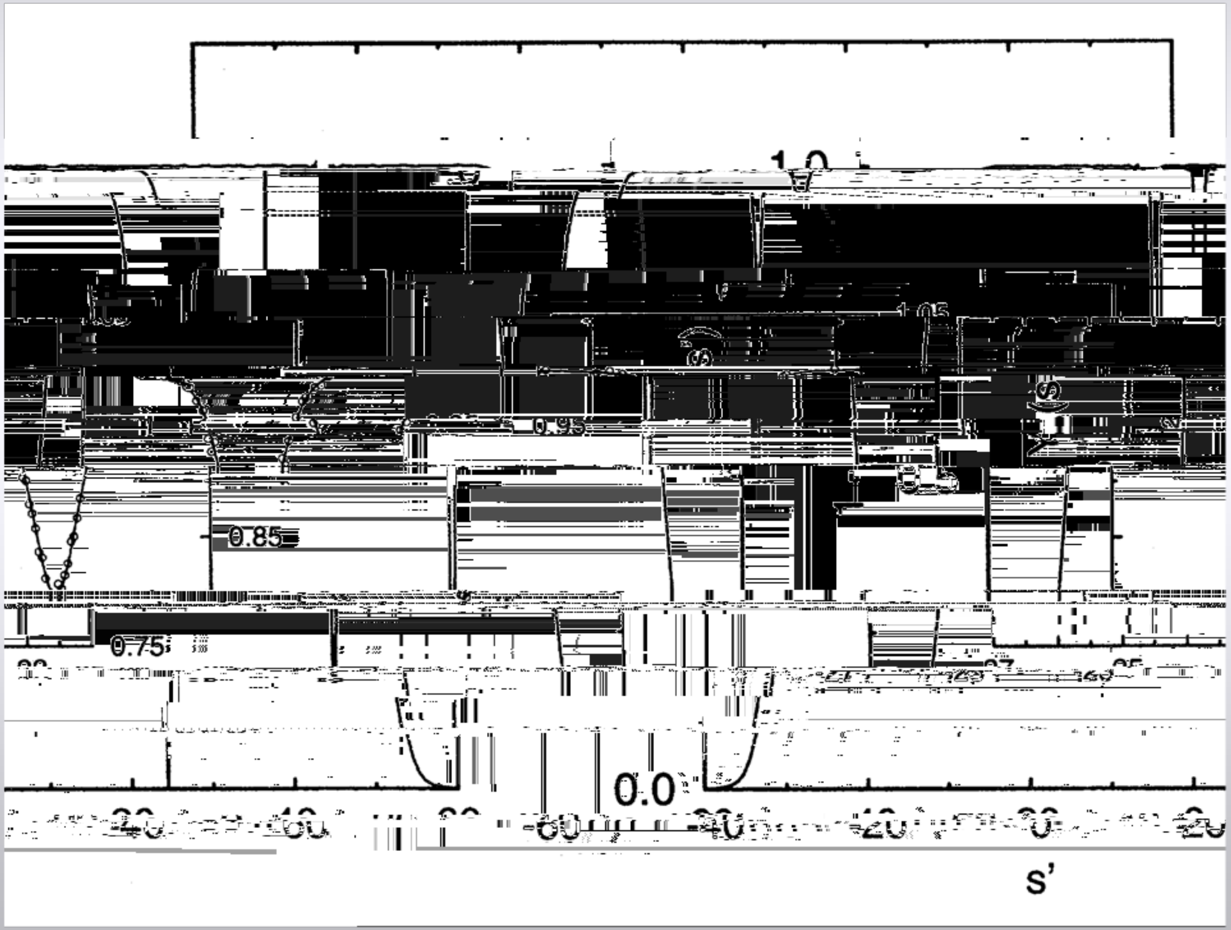
- For $e^{-2\pi^2/\kappa} \ll 1$ semiclassical analysis (Canali et al '95):

[redacted]

$$Y_2(x,x') \equiv \delta(x-x') - \frac{\langle\rho(E_x)\rho(E_{x'})\rangle - \langle\rho(E_x)\rangle\langle\rho(E_{x'})\rangle}{\langle\rho(E_x)\rangle\langle\rho(E_{x'})\rangle}$$

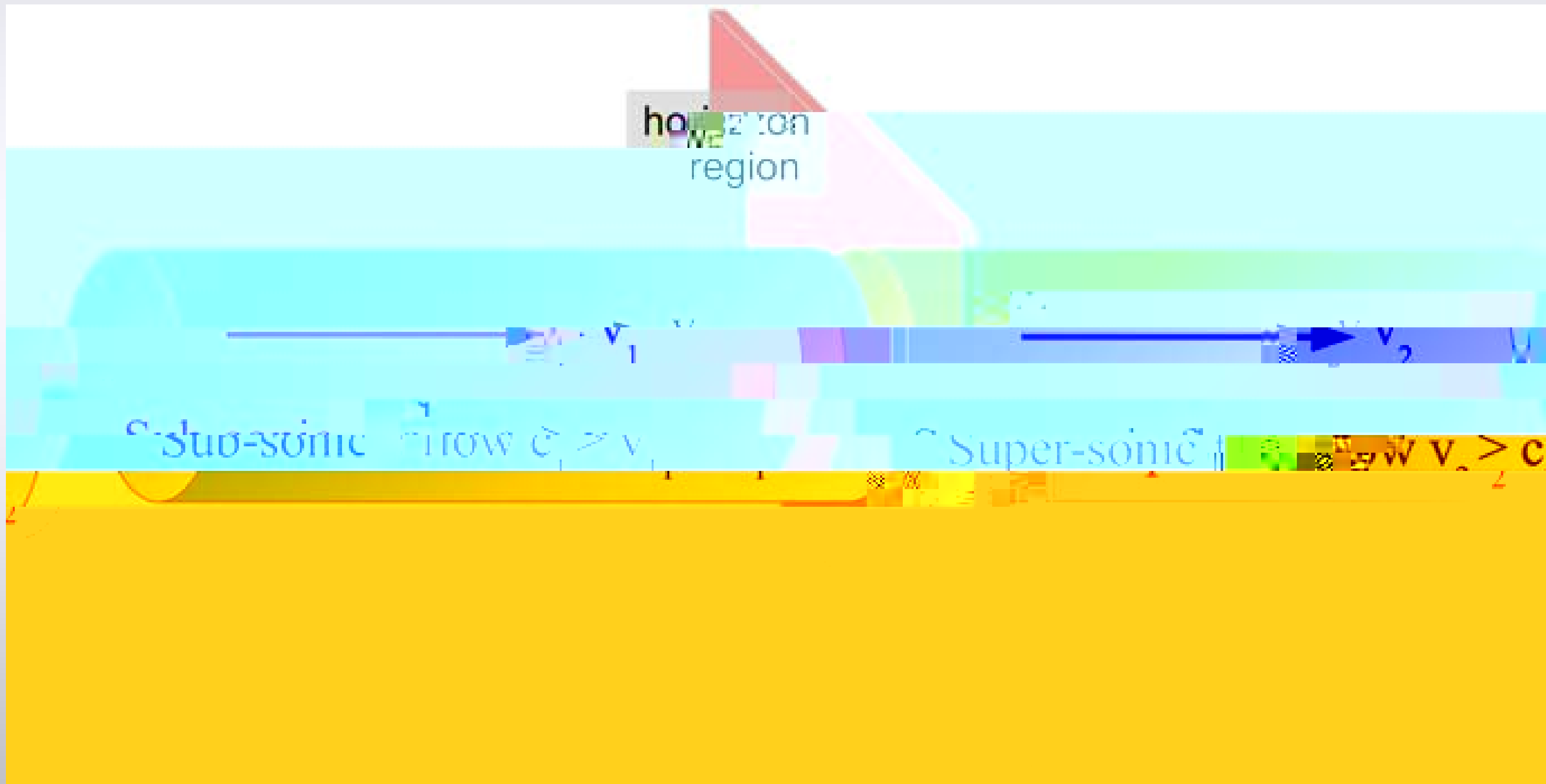
Weakly Confined Invariant Ensemble

- **Numerical check** (Canali et al '95):



Acoustic Black-Hole

- Fluid pushed to move faster than it's speed of sound:



Hawking radiation

- Prediction: a Black Hole radiates particles with an exact thermal (*Black-Body*) spectrum
- Solid result due only to horizon (kinematical)
- Different ways to understand it:
  - Pair production close to horizon
  - Red-shifting of last escaping modes
  - Casimir effect
  - Bogoliuobov overlap of positive frequency modes close to the horizon and at infinity
  - …

QFT in Curved Space-Time

- Field quantization is basis-dependent:

[redacted]

Plane waves

$\Rightarrow$ vacuum depends on the observer: [redacted]

- For a different coordinate system:

$$\phi(x) = \sum_i \left[\tilde{a}_i \tilde{f}_i(x) + \tilde{a}_i^\dagger \tilde{f}_i^*(x) \right]$$

$\Rightarrow$ [illegible]

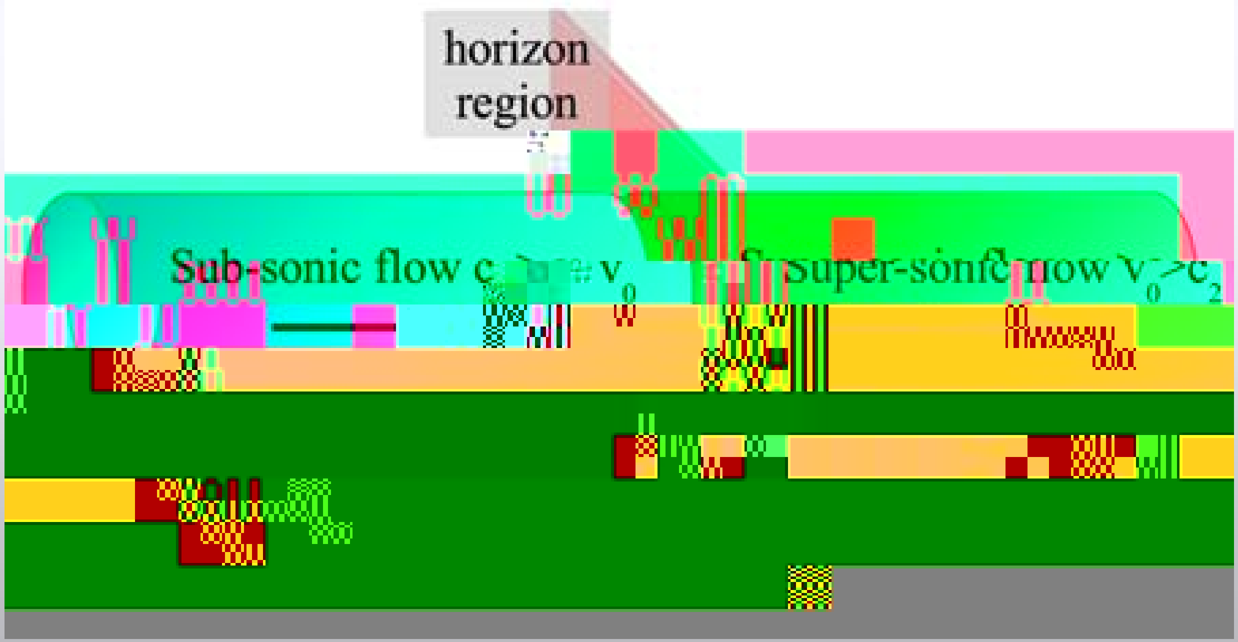
Hawking Radiation

- If

$\Rightarrow$

Acoustic BH in BEC

- Cool system → Bose-Einstein Condensate
- Keep stream velocity constant & change speed of sound
- Effective dynamics is 1-D



Effective Gravity in fluids

- Low-Energy excitations (phonons) propagate on top of bulk stream
-

2-Point Correlator

- In flat space: $\langle \delta\phi(x,t)\delta\phi(x',t')\rangle \propto \ln(\Delta u^+ \Delta u^-)$

$$u^\pm \equiv t \pm \int \frac{\mathrm{d}x}{c \mp v}$$

← **Light-Cone coordinates**

and for the density: $\langle \delta\rho(x,0)\delta\rho(x',0)\rangle \propto \frac{1}{(x-x')^2}$

- Around the black hole: $u^- \to \tilde{u}^- = \frac{1}{\kappa} e^{-\kappa u^-}\,\mathrm{sgn}(x)$

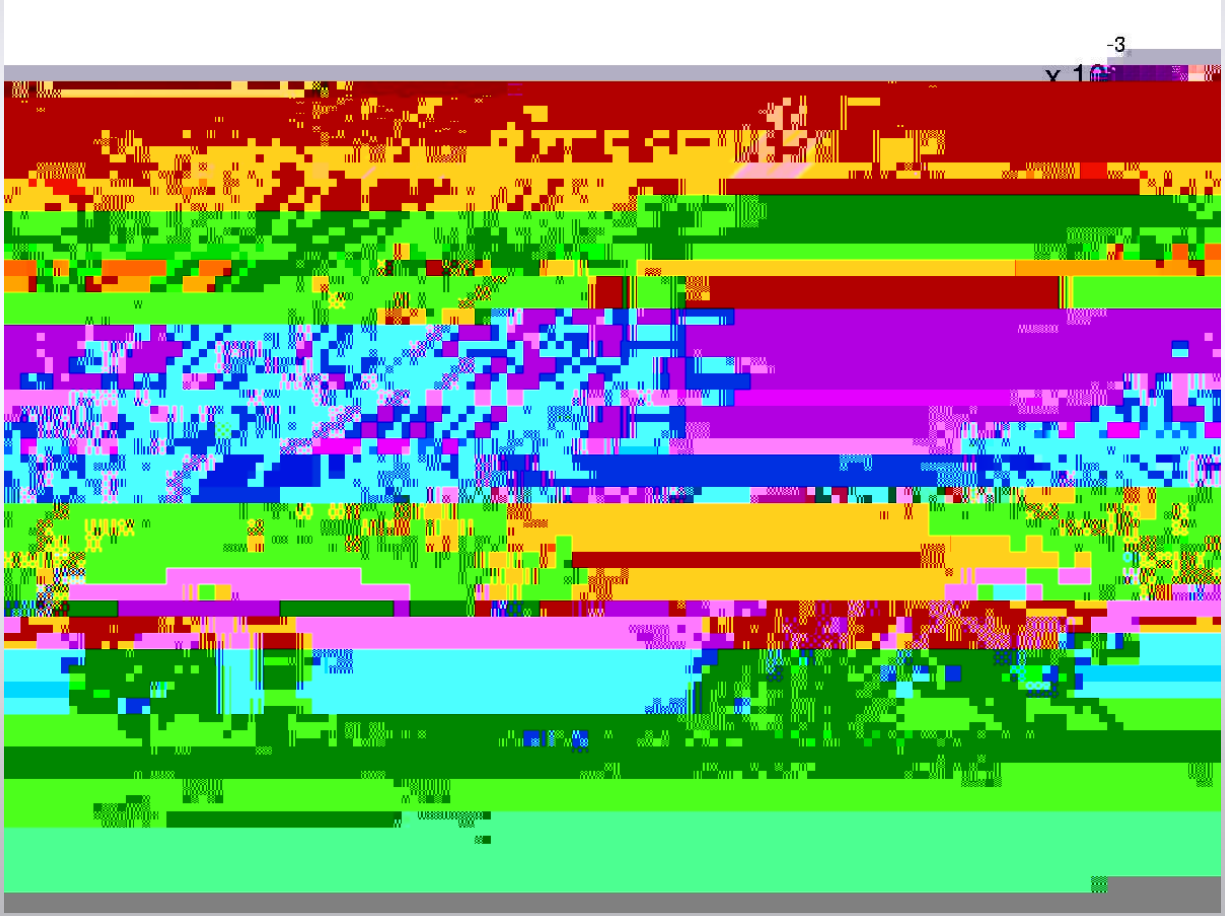
2-Point Correlator in curved metric

- Same non-local correlation as RME for $c_{r,l} = v \pm v/2$

(<u>except</u> for the oscillatory term...)

Numerical check

- Field theory prediction checked against ab-initio numerical simulation (Carusotto et al. ’08)



- BEC system has non-local signature

- Low-Energy description in terms of free field in curved metric with horizon

Effective Theory for RME

$$\mathcal{L} = -\beta \sum_{n>m} \ln |E_n - E_m| + \sum_n V(E_n)$$

- Energy eigenvalues
  $\rightarrow$ coordinates of interacting particles
  (fermions $\Leftarrow$ level repulsion)

- Parametric evolution of RME
  $\rightarrow$ time coordinate

- Eigenvalue distribution
  $\rightarrow$ ground state configuration of 1D quantum model

Luttinger theory for RME

Unfolding:

$\rho_0 = 1$

- Two-Point function (Kravtsov et al. '00):

- In flat space: $\langle \Phi(x,t)\Phi(x',t')\rangle \propto \ln\left(\Delta x^2 + \Delta t^2\right)$

2-Point Function
for Gaussian RME
(K=1: Unitary)

Non-invariant Critical Ensemble

- Critical Random Banded Matrix (Multifractal spectrum)

$$P(\mathbf{H}) \propto \mathrm{e}^{-\sum_{n,m} A_{nm}\, |H_{nm}|^2} \qquad A_{nm} = 1 + \frac{(n-m)^2}{[illegible]}$$

- Thermal effective Luttinger Theory (Kravtsov & Tsvelik-2001)

$g*$: critical conductance

[illegible]

$$[illegible],\ T \equiv [illegible]\ Y_2(x,x') = T^2\, \frac{[illegible]}{[illegible]}$$

Thermal Field Theory

- Diagonal part of 2-point function

$$T^2 \frac{\sin^2[\pi T(x-x')]}{\sinh^2[\pi T(x-x')]} Y_2(x,x') =$$

common to

ü weakly confined invariant ensemble

ü Lorentzian banded matrix ensembles

ü Standard thermal field theory

- How to generate the non-translational invariant part?

Luttinger theory in curved metric

$$P(\mathbf{H}) \propto e^{-\mathrm{Tr}V(\mathbf{H})} \qquad V(E) \overset{|E|\to\infty}{\simeq} \kappa \ln^2 |E|$$

- BEC system taught us that metric with horizons gives non-local correlation function

$$S[\Phi] = \frac{1}{8\pi K} \int \mathrm{d}^2\xi \sqrt{-g(\xi)}\, g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$$

- In 1+1 D any horizon metric can be approximated by Rindler line element

$$g \equiv \det g^{\mu\nu}, \quad \mathrm{d}s^2 = g_{\mu\nu} \mathrm{d}\xi^\mu \mathrm{d}\xi^\nu$$

$$\mathrm{d}s^2 = -y^2 \mathrm{d}t^2 + \frac{1}{y^2} \mathrm{d}y^2$$

Horizon at $y=0$

- Let's choose: $y \equiv \sinh(\kappa x)$

Luttinger theory in Rindler space

- Far from the origin:

Luttinger Liquid in Rindler Space

- Remind two-Point function:

$$Y_2 = \frac{1}{\pi^2}\langle \partial_x \Phi(x) \partial_{x'} \Phi(x') \rangle$$

$$+ \dots \quad - \frac{A_K^2}{2\pi^2} \cos(2\pi(x-x'))\langle \mathrm{e}^{\mathrm{i}2\Phi(x)} \mathrm{e}^{-\mathrm{i}2\Phi(x')} \rangle$$

- With the new coordinates: $\left(\bar{x} = \frac{\mathrm{e}^{\kappa |x|}}{2\kappa} \mathrm{sgn}(x) \right)$

$$\langle \Phi(x)\Phi(x') \rangle \overset{|x|,|x'|\gg 1}{\propto} \begin{cases} \ln\left[\frac{2}{\kappa}\sinh\frac{\kappa(x-x')}{2}\right], & x\,x' > 0 \\ \ln\left[\frac{2}{\kappa} \dots \frac{\dots}{2}\right], & x\,x' < 0 \end{cases}$$

Luttinger Liquid in Rindler Space

- We recover exactly the RME correlation (K=1):

$$Y_2^a(x,x') = \frac{\kappa^2}{4\pi^2}\frac{\sin^2[\pi(x-x')]}{\cosh^2[\kappa(x+x')/2]}, \quad \text{for } x, x' < 0$$

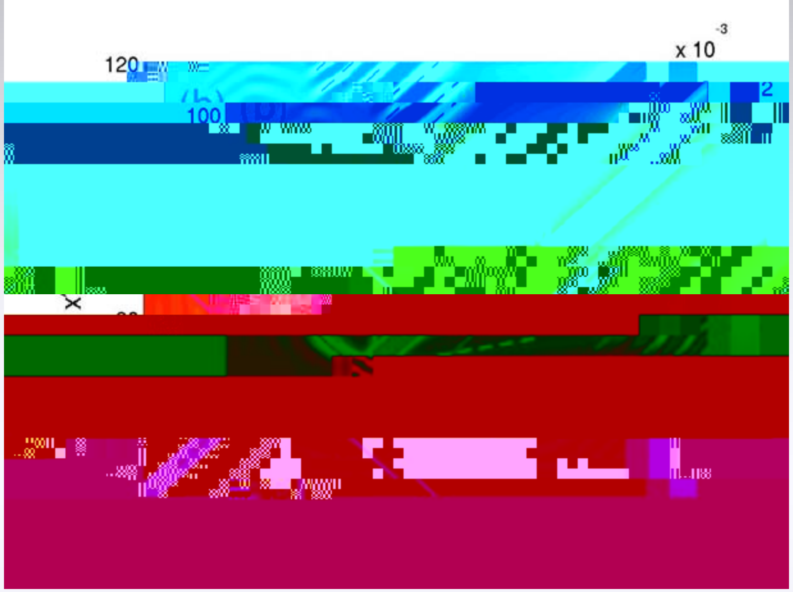
(Anomalous: non-translational invariant)

(Normal: translational invariant)

Summing up… (part 1)

- Luttinger Liquid predicts oscillatory term in correlator

- Possible to detect them in a BEC in Tonks-Girardeau regime



[redacted]

- We reproduced the asymptotic 2-point function in a Luttinger Liquid in curved space-time description
- Curved metric with horizons → Hawking radiation
- Equivalence with BEC system (oscillatory term)
- Underlying integrable model as interesting probe for emerging Quantum Gravity (trans-planckian problem)
- Many unresolved questions (microscopical derivation?); nature of thermal bath