The $O(n)$ model on random lattices of all topologies

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1 - $O(n)$ model

$$W_k^{(g)}(x_1,\dots,x_k) = \sum \frac{1}{\text{number of automorphisms}} \frac{t^v t_3^{n_3}\cdots t_d^{n_d}\, n^L z}{x_1^{j_1+1}\cdots x_k^{j_k+1}}$$

ranges over connected genus g discrete surface with k boundaries, built with:

- v vertices
- n_j j-gons ($j \geq 3 \geq d$, d fixed but arbitrary)
- triangles carrying a piece of path forming exactly L loops
- a marked j_i-gon ($j_i \geq 1$) with a marked edge as i-th boundary ($1 \leq i \leq k$)

For n N, it admits a representation as a formal hermitian matrix model [1, 6]:

$$d\lambda(M, A_1,\dots,A_n) = \underbrace{dM\, dA_1 \cdots dA_n}_{\text{Lebesgue measure on } H_N^{1+n}} \exp\left(-\frac{N}{t}\mathrm{Tr}\left(V(M) + \sum_{i=1}^{n}\left(\frac{1}{2z} + M\right)A_i^2\right)\right)$$

$$\left\langle \prod_{x_i \in K} \mathrm{Tr}\,\frac{1}{x_i - M}\right\rangle_c = \sum_{g=0}^{\infty}\left(\frac{N}{t}\right)^{2-2g-k} W_k^{(g)}(K)$$

$$V(x) = \frac{x^2}{2} - \sum_{j=3}^{d}$$