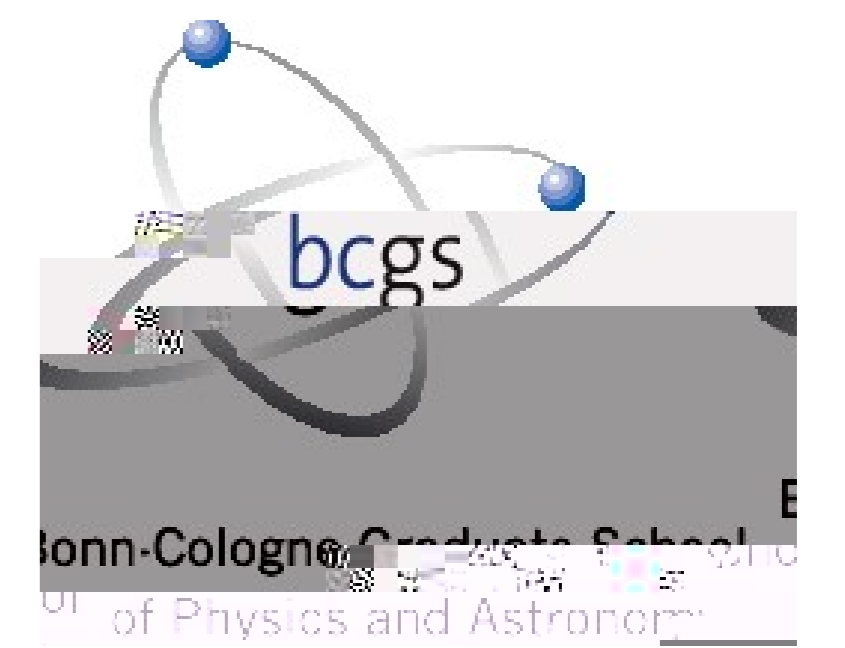


# Local level statistics

*Free probability meets supersymmetry*

Artur Swiech & Martin Zirnbauer  
University of Cologne, Institute for Theoretical Physics  
swiech@thp.uni-koeln.de



## Abstract

Our goal is description of local eigenvalue statistics of invariant random matrix models. We consider  $N \times N$  random matrices, governed by a confining analytic potential  $V(H)$ , and study the characteristic function in the  $N \rightarrow \infty$  limit. We use supersymmetry method to obtain behavior of many-point correlation functions. An important lemma, existence and uniqueness of supersymmetric Laplace Transform, has been proven for the functions in question. This technique is a powerful method of determining universality classes in correlations of eigenvalues.

## Introduction

Invariant random matrix ensembles:

$$d_N(H) \propto e^{-N \text{Tr} V(H)} dH: \quad (1)$$

Characteristic function:

$$(K) = \int e^{\text{Tr} KH} d(H): \quad (2)$$

For analytic and convex  $V$  we have saddle point equation [1]:

$$Q^{-1} + R(Q) = \text{zld}_{pq} \quad (3)$$

where  $Q$  is a rank  $(p|q)$  supermatrix argument of lifted  $\hat{\Delta}(Q)$  required by supersymmetry method and  $R$  is a free probabilistic R-transform. Correlation functions may be retrieved from  $\hat{\Delta}$  by analog of Laplace Transform:

$$\int \frac{\prod_{b=1}^q \text{Det}(W_{1;b} H)}{\prod_{a=1}^p \text{Det}(W_{0;a} H)} d_N(H) \propto \int \text{SDet}^N(Q) \hat{\Delta}(Q) e^{-\text{STr} w Q} DQ \quad (4)$$

## Objectives

1. Establish existence and uniqueness of Laplace Transform
2. Analyze singularities of R-transform
3. Obtain minimal requirements for the formalism
- 4.