Department of Economics and Finance

EXPONENTIAL TIME TRENDS
IN A FRACTIONAL INTEGRATION MODEL

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Abstract

This paper introduc

1. Introduction

It is common practice in applied work to allow for simple linear deterministic trends when modelling standard economic and financial series (Bhargava, 1986; Stock and Watson, 1988; Schmidt and Phillips, 1992). However, some of them appear to be characterised by exponential growth as in the case of compound interest. An exponential growth trend can be captured by taking logs of the series of interest and regressing it against a constant and a linear trend. However, fitting a linear trend with a constant growth rate is in most cases too restrictive.

where parameters (the intercept, the time trend coefficient and its exponent respectively); in addition, x_t is assumed to be an integrated process of order d, i.e.,

$$(1 - B)^d x_t = u_t, \qquad t = 1, 2, \ldots, \qquad (2)$$

where d can be any real value, B is the backshift operator, i.e., $B^k x_t = x_{t-k}$, and thus u_t is an I(0) process, more precisely a covariance-stationary one with a spectral density function that is positive and bounded at the zero frequency.

We test the null hypothesis:

$$H_o : d = d_o, \qquad (3)$$

for any real value d_0 in the model given by (1) and (2) by ,

for example between 0 and 2, with 0.01 increments. Under the null hypothesis (3), the model given by (1) and (2) becomes:

$$\tilde{\ } = \tilde{1} + (\tilde{\ }) + \ , \quad = 1,2,\ldots, \qquad (4)$$

where

$$\tilde{y}_t = (1 - B)^{d_o} y_t,$$

and

$$\tilde{1}_t = (1 - B)^{d_o} 1, \quad \text{and} \quad \tilde{\ } = (1 - \ ) \ (\ ) .$$

Since the value of set, one

basis of (5), the null H_o (3) will be rejected against the alternative H_a: d ¸ d_o if $R > c^2_{1,a}$,

with Prob ($c^2_1 > c^2_{1,a}$) = a. It is easy to see that this result holds for any value of   in the interval (0, 1). Specifically, Robinson (1994) used the following regression model

=

increase as the sample size increases, which is consistent with the asymptotic behaviour of the test.

TABLES 1 AND 2 ABOUT HERE

Table 2 is similar to Table 1 but reports the results based on the t_3-Student distribution for the error term. Once again the sizes are higher than the 5% level and higher values are observed against departures of the form $d < d_o$. The rejection frequencies are also higher for this type of departures, and even for small ones the rejection frequencies are relatively high.

4. Three Empirical Applications

, d = 1.28 and the 95% confidence interval being given by (1.17, 1.42), with

, both being statistically significant. Thus, the unit

, which indicates the presence

of a concave time trend in the data.

Table 4 has the same layout as the previous one but concerns the S&P500 stock market index. The estimates of d now range between 0.91 and 1.24 and the lowest statistic

0.97 (0.92, 1.24). Thus, a linear time trend with a unit root seems to be a plausible hypothesis; this is consistent, for t >2, with a random walk model with an intercept, and thus with the Efficiency Market Hypothesis (EMH) in its weak form (Fama, 1970).

Finally, Table 5 reports the corresponding results for the US Consumer Price Index. In this case d is much higher than 1 (specifically, 1.44), with a confidence interval given by (1.38, 1.52), and thus the unit root null hypothesis is rejected in favour of d > 1; also, the estimate of implies a convex time trend.

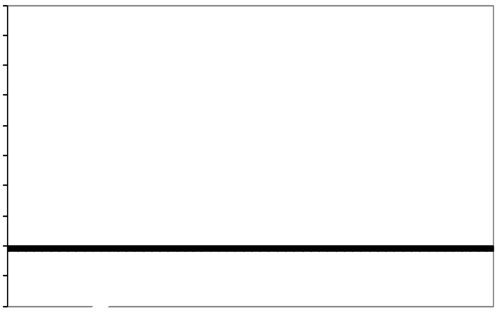
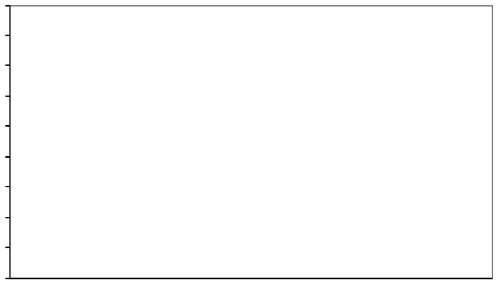
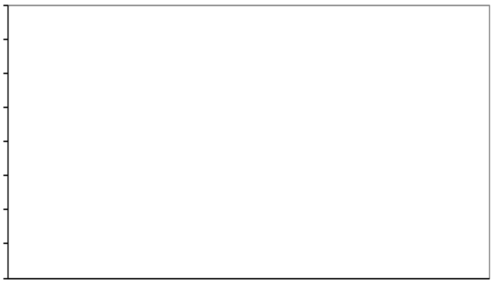
5. Conclusions

This paper puts forward a modelling and testing framework that allows for exponential deterministic trends in a fractional integration context. The Montecarlo simulations carried out to examine the properties of the proposed test indicate that it performs well in finite samples. As an illustration, the

Figure 1: Realisations from Equations (1) and (2) with d = 0.25

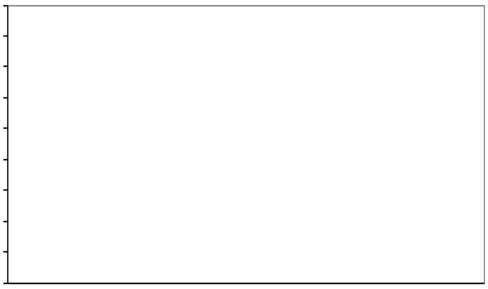
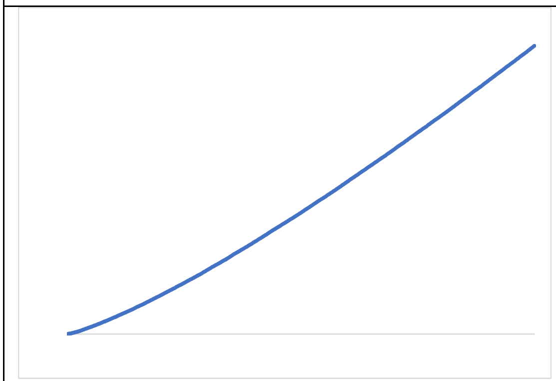
	0.50
0.75	1.00
1.25	1.50

Figure 2: Realisations from Equations (1) and (2) with d = 0.50

	0.50
	
0.75	1.00
	
1.25	1.50

Note: We generate Gaussian series with T = 1000, and then produce the realisations of y_t in (1) and (2) with d = 0.50.

Figure 3: Realisations from Equations (1) and (2) with d = 0.75

	0.50
	
0.75	1.00
	
1.25	1.50
	

Note: We generate Gaussian series with T = 1000, and then produce the realisations of y_t in (1) and (2) with d = 0.75.

Figure 4: Realisations from Equations (1) and (2) with d = 1.00

0.25	0.50
0.75	1.00
1.25	1.50

Note:

Figure 5: US Real GNP Per Capita

Table 3

Table 4: Estimated coefficients for the log of US real GNP per capita

	d	95% band	-value)	(t-value)	Stastistic
0	0.96	(0.93, 1.02)	41.119 (0.171)	---	0.227
0.10	0.96	(0.93, 1.02)	42.633 (0.143)	49.836 (0,16)	0.219
0.20	0.97	(0.92, 1.23)	52.366 (0.399	39.924 (0.31)	0.203
0.30	0.97	(0.92, 1.23)	54.121 (0.70)	37.914 (0.56)	0.239