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# **LONG-RUN TRENDS AND CYCLES IN US HOUSE PRICES**

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## **Abstract**

This paper analyses US nominal house prices at an annual frequency over the period from 1927 to 2022 by means of a very general time series model. This includes both a (linear and non-linear) deterministic and a stochastic component, with the latter allowing for fractional orders of integration at both the long-run and the cyclical frequencies. The results are heterogeneous depending on the model specification and on whether or not the series have been logged. Specifically, a linear model appears to be more appropriate for the logged data whilst a non-linear one appears to be a better fit for the original ones. Further, the order of integration at the zero or long-run frequency is much higher than at the cyclical one. The former is in fact around 1 in all spe 1 in al

## **1. Introduction**

House prices are a key variable whose fluctuations can have a significant impact on both the real and the financial sectors of the economy, as documented, among others, by Case et al. (2005), Davis and Heathcote (2005), Leamer (2007), Attanasio et al. (2011), Carroll et al. (2011), Funke and Paetz (2013), Chen et al. (2018). Their crucial importance became even more apparent as a result of the global financial crisis (GFC) of 2007-08. This originated from the US housing market, where the issuance of sub-prime mortgages had become widespread and led to a housing bubble and serious financial turmoil when it eventually burst (see, e.g., Shiller, 2007). Consequently, numerous empirical studies have been carried out to understand the behaviour of house prices. Broadly speaking, two main approaches have been followed in the literature for this purpose, the first focusing on their drivers, the second on their stochastic properties. Among studies belonging to the first category, Capozza and Helsely (1989, 1990)

Granger and Joyeux, 1980 and Hosking, 1981) is much more general, since it is not based on the dichotomy between  $I(0)$  stationary and  $I(1)$  non-stationary series, which is very restrictive. Instead the differencing parameter  $d$  is allowed to take any real value, including fractional ones. This approach encompasses a wide range of stochastic behaviours, including the unit root case, and provides evidence on whether or not the series of interest is mean-reverting (and thus on whether exogenous shocks have permanent or transitory effects) and on its degree of persistence. It has been used in some

a general form as in Bloomfield (1973) is allowed in the error term. This framework is applied to analyse US nominal house prices at an annual frequency over the period from 1927 to 2022.

The layout of the paper is the following. Section 2 outlines the modelling framework. Section 3 describes the data and presents the empirical results. Section 4 offers some concluding remarks.

## 2. The Econometric Model

The model estimated in this study is more general than those used in the previous literature on house prices. Specifically, it includes both a deterministic and a stochastic component, with the latter allowing for fractional degrees of integration at both the long-run and cyclical frequencies.

The deterministic part of the model is specified as follows:

$$y(t) = \beta_0 + \beta_1 t + \epsilon(t) \quad (1)$$

where  $y(t)$  stands for house prices (either the original or the logged series), and  $f$  is a function that can be linear, for instance including an intercept and a linear time trend, (Bhargava, 1986, Schmidt and Phillips, 1992) as in the following equation:

**B**

Hamming (1973) and Smyth (1998) provided a detailed description of these polynomials, whilst Bierens (1997) and Tomasevic and Stanivuk (2009) argued that it is possible to approximate highly non-linear trends with rather low degree polynomials. If  $m = 0$  the model contains an intercept, and if  $m = 1$ , it becomes non-linear - the higher  $m$  is, the less linear the approximated deterministic component becomes.

Concerning the stochastic terms,  $x(t)$  in (1) is assumed to be a process characterised by two orders of integration, one for the long-run or zero frequency, which captures possible stochastic trends, and the other for the cyclical structure of the data. More precisely,  $x(t)$  is defined as follows:

$$\mathbb{C}^{-}$$

$\epsilon_t$  is a white noise process, its spectral density function is given by:

$$f_{\epsilon}(\omega) = \frac{\sigma^2}{2\pi}$$

According to Bloomfield (1973), the log of the above expression can be well approximated by Eq. (5) when  $p$  and  $q$  are small values, and thus it does not require the estimation of as many parameters as in the case of ARMA models. In addition, Bloomfield (1973) model has the advantage of being stationary for all its values (see Gil-Alana, 2004).

Let us now consider further Eq. (4). Note that the first polynomial can be expanded for any real value  $d_1$  as

$$1 - \theta(B) = (1 - \theta_1 B + \theta_2 B^2 - \dots)$$

In this context,  $d_1$  indicates the degree of persistence of the series in relation to the long-run or zero frequency. Thus, if  $d_2 = 0$  in Eq. (4),  $x(t)$  can be expressed as

$$x(t) = \frac{1}{(1 - \theta_1 B + \theta_2 B^2 - \dots)} \epsilon_t$$

and the higher the value of  $d_1$  is, the higher is the degree of dependence between the observations. Moreover, if  $d_1$  is positive,  $x(t)$  displays the property of long memory since in that case its spectral density function becomes

$$f_x(\omega) = \frac{1}{(1 - \theta_1 B + \theta_2 B^2 - \dots)^2}$$

- iii) long memory, though covariance stationary processes, if  $0 < d_1 < 0.5$ ,
- iv) 1/f noise, if  $d_1 = 0.5$ ,
- v)  $d_1 < 1$ ,
- vi) unit roots, if  $d_1 = 1$ ,
- vii) explosive processes, if  $d_1 > 1$ .

Next we focus on the cyclical structure of  $x(t)$  which is captured by the second polynomial in (4). Gray et al. (1989) showed that, by  $\dots$  is polynomial can be expressed in terms of the orthogonal Gegenbauer terms  $\dots$ , such that for all

where  $\dots$  can be defined recursively as:

and

This type of process was introduced by Andel (1986), and authors such as Gray et al. (1989, 1994), Giraitis and Leipus (1995), Chung (1996a, 1996b), Gil-Alana (2001), Dalla and Hidalgo (2005), Caporale and Gil-Alana (2013) and others subsequently used it to analyse time series data.

3. Data DescBT714 12 Tf1 0 0 1 411.67 302.45 Tm0 g0 G(-)JTJETQ0.000008871 0 595.32 841.92 r



Figure 1 displays time series plots of the original series, its logged transformation, and the first differences of both. It can be seen that the series in levels, whether logged or

specification is chosen on the basis of the statistical significance of the estimated coefficients. We report the results for both the original and log-transformed data in levels.

**TABLES 1 AND 2 ABOUT HERE**

Table 2 displays the estimated parameters from the selected model for each of the two series. The time trend is statistically significant in both cases with a positive coefficient, and the estimates of  $d$  are 0.85 for the original data and 0.97 for the log-transformed ones. However, the confidence intervals imply that the unit root null hypothesis (i.e.,  $d = 1$ ) cannot be rejected for either series.

Next, we consider a non-linear specification with Chebyshev polynomials in time. Specifically, the estimated model is now the following:

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where  $d$

while



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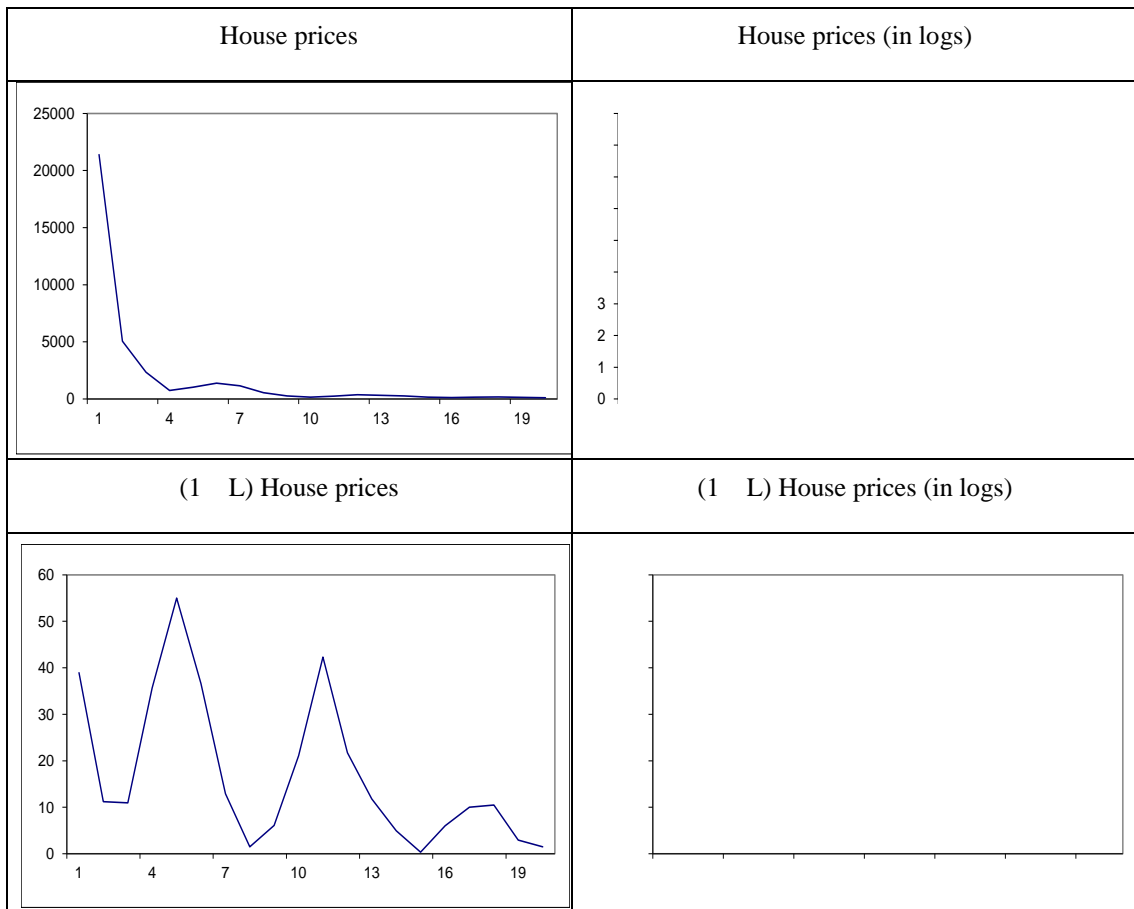


**Figure 2: Correlograms of the series**

House prices

House

**Figure 3: Periodograms of the series**



**Note:** The displayed

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**Table 1: Estimates of d at the long-run frequency with a linear trend model**

Series	No deterministic terms	With an intercept	With an intercept and a linear trend
Original data	0.83 (0.62, 1.66)	0.78 (0.65, 1.47)	<b>0.85 (0.57, 1.45)</b>
Logged data	0.78 (0.55, 1.28)	0.97 (0.80	

