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BITCOIN RETURNS

AND THE FREQUENCY OF DAILY ABNORMAL RETURNS

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Abstract

This paper investigates the relationship between Bitcoin returns and the frequency of daily abnormal returns over the period from June 2013 to February 2020 using a number of regression techniques and model specifications including standard OLS, weighted least squares (WLS), ARMA and ARMAX models, quantile regressions, Logit and Probit regressions, piecewise linear regressions and non-linear regressions. Both the in-sample and out-of-sample performance of the various models are compared by means of appropriate selection criteria and statistical tests. These suggest that on the whole the piecewise linear models are the best but in terms of forecasting accuracy they are outperformed by a model that combines the top five to produce ñconsensusò forecasts. The finding that there exist price patterns that can be exploited to predict future price movements and design profitable trading strategies is of interest both to academics (since it represents evidence against the EMH) and to practitioners (who can use this information for their investment decisions).

Keywords: *cryptocurrency, Bitcoin, anomalies, abnormal returns, frequency of abnormal returns, regression analysis*

JEL classification: G12, G17, C63

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evidence is mixed: some papers find price reversals after abnormal price changes (Bremer and Sweeny, 1991; Larson and Madura, 2001), whilst others detect momentum effects (Schnusenberg and Madura, 2001; Lasfer et al., 2003). In the specific case of the cryptocurrency markets, Chevapatrakul and Mascia (2019) estimated a quantile autoregressive model and concluded that days with extremely negative returns are likely to be followed by periods characterised by weekly positive returns as Bitcoin prices continue to rise. Caporale and Plastun (2019) used a variety of statistical tests and trading simulation approaches and found that after one-day abnormal returns price changes in the same direction are bigger than after ñnormalò days (the so-called momentum effect). Caporale et al. (2019) provided evidence on the role played by the frequency of overreactions. Qing et al. (2019) applied DFA and MF-DFA methods and found momentum effects in Bitcoin and Ethereum prices after abnormal returns. Momentum effects were also detected by Panagiotis et al. (2019) and Yukun and Tsyvinski (2019). The present study extends the previous one by Caporale et al. (2019) as detailed below.

3. Methodology

The selected sample includes daily and monthly BitCoin data over the period 06.2013-02.2020. The data source is CoinMarketCap:

https://coinmarketcap.com/currencies/bitcoin/. For forecasting purposes two subsamples are created, namely 01.06.2013-30.12.2018 and 01.01.2019-28.02.2020 at the daily frequency, and June 2013-December 2018 and January 2019-February 2020 at the monthly frequency; various models are estimated over the first subsample, forecasts are then generated in each case for the second subsample using the estimated parameters and their accuracy is evaluated by means of various statistical criteria.

treating all observations equally they are weighted to increase the accuracy of the estimates.

To obtain further evidence an ARMA(p,q) model is also estimated (4):

(4)

where ï BitCoin log returns in month *t;*

a_0 ï constant;

${}_{t\ i}$; ${}_{t\ j}$ï coefficients the lagged log returns and random error terms respectively;

ï random error term at time *t*;

This is a special case of an ARIMA(p,d,q) specification with d=0, which is appropriate in our case since all series are stationary, as indicated by a variety of unity root tests which imply that differencing is not required (the test results are not reported for reasons of space but are available from the authors upon request).

Next, in order to improve the basic ARMA(p,q) specification exogenous variables are added, namely the frequency of negative and positive one-day abnormal returns in (5) and Delta in (6), to obtain the following ARMAX(p,q,2) and ARMAX(p,q,1) models:

$$Y_t \quad a_0 \quad {}^{p}_{i\ 1} \quad {}_{t\ i}Y_{t\ i} \quad {}^{q}_{i\ 0} \quad {}_{t\ i\ \ t\ i} \quad a_1F_t \quad a_2F_t \qquad (5)$$

$$Y_t \quad a_0 \quad {}^{p}_{i\ 1} \quad {}_{t\ i}Y_{t\ i} \quad {}^{q}_{i\ 0} \quad {}_{t\ i\ \ t\ i} \quad a_1Delta_t \qquad (6)$$

A non-parametric method not requiring normality is also used; specifically, quantile regressions are run to estimate the conditional median instead of the conditional mean. More precisely, the quantile regression model for the -th quantile is specified as follows (7-8):

(7)

$$Y_t \quad a_0(\ ) \quad a_1(\ )Delta_t \quad {}_{t}(\ ) \qquad (8)$$

where ï the -th quantile and *;*

Next, Probit and Logit regression models are estimated. These are specific cases of binary choice models that provide estimates of the probability that the dependent variable will take the value 1. In a Logit regression, it is assumed that $P\{y \quad 1|x\} \quad f(z)$

Information criteria, namely AIC (Akaike, 1974) and BIC (Schwarz, 1978), are used to select the best model specification for Bitcoin log returns. To compare the forecasting performance of different models various measures such as the

The OLS and WLS regression results are reported in Table 1. Models 1 and 2 are the standard OLS regressions given by (2) and (3), whilst models 1.1 and 2.1 are the WLS ones, where the weights are the inverse of the standard error for each observation used.

Table 1: Regression analysis results: BitCoin log returns

Parameter	Model 1	Model 1.1	Model 2	Model 2.1
	Delta	Delta	Frequency of negative and positive abnormal returns as separate variables	Frequency of negative and positive abnormal returns as separate variables
	0.0901 (0.000)	0.0777 (0.000)	0.0650 (0.024)	0.0626 (0.023)
Coefficient on abnormal returns ( )	0.0953 (0.000)	0.0868 (0.000)	-	-
Coefficient on the frequency of negative abnormal returns	-		-0.0904 (0.000)	-0.0849 (0.000)
Coefficient on the frequency of positive abnormal returns	-		0.0993 (0.000)	0.0916 (0.000)
R^2	0.7721	0.7652	0.7767	0.7722

p-valu3 Tm0 ga05 424.87 0.48001

between the actual and estimated values suggests that Bitcoin is over- or under-valued, and therefore that it should be sold or bought till the observed difference disappears, at which stage positions should be closed.

The estimates from the selected ARMA(p,q) models on the basis of the AIC and BIC information criteria, namely ARMA(2,2) and ARMA(3,3), are presented in Table 2. As can be seen, although most coefficients are significant, the explanatory power of these models is rather low.

Table 2: Parameter estimates for the best ARMA models

Parameter	Model 3: ARMA(2,2)	Model 4: ARMA(3,3)
a_0	0.0516(0.2103)	0.0513(0.1887)
t 1	0.3486(0.006)	-
t 2	-0.7381(0.000)	-0.3874(0.000)
t 3	-	-0.6209(0.000)
t 1	-0.3418(0.000)	-
t 2	1.000(0.000)	0.5790(0.000)
t 3	-	0.6487(0.000)
R^2	0.0562	0.0373
Log Likelihood	-12.3733	-13.3259
Model Standard Error	0.2831	0.2885
AIC	36.7466	38.6518
BIC	49.9748	51.8800

This table presents the coefficient estimates and p-

Table 3: Estimated parameters for the ARMAX models: regressors F and F

Parameter	Model 5 ARMAX(1,1,2)	Model 6 ARMAX(2,2,2)	Model 7 ARMAX(3,3,2)
a_0	0.0710(0.0674)	0.0678(0.0193)	0.0653(0.0185)
t 1	0.9488(0.000)	-1.3021(0.000)	-
t 2	-	-0.7734(0.000)	-0.1899(0.0932)
t 3	-	-	-0.8078(0.000)
t 1	-0.8963(0.000)	0.06834(0.000)	-
t 2	-	1.0000(0.000)	0.3585(0.000)
t 3	-	-	0.8009(0.000)
a_1	0.0996(0.000)	0.1020(0.000)	0.0973(0.000)
a_2	-0.0927(0.000)	-0.0936(0.000)	-0.0886(0.000)

t 1

Log Likelihood	37.2054	33.5500
Model Standard Error	0.1055	0.1115
AIC	-68.4109	-62.9594
BIC	-61.7968	-58.5500

Table 7: Estimated parameters for the

a_6	-	-	-0.0006(0.005)
b	0.0511(0.008)	0.0590(0.003)	0.0481(0.000)
c	-0.0589(0.030)	-0.0709(0.011)	-0.0472(0.002)
p	1.2753(0.000)	1.1776(0.000)	1.4688(0.000)
q	1.1609(0.000)	0.9531(0.000)	

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Appendix B

Table B.1: Predicted vs actual values over the period 01.2019-02.2020

Period	01.2019	02.2019	03.2019	04.2019	05.2019	06.2019	07.2019	08.2019	09.2019	10.2019

Appendix C

Table C.1: Forecasting accuracy tests

Parameter	Root Mean Square Error (RMSE)	Mean Absolute Error (MAE)	Mean Percentage Error (MPE), %	Mean Absolute Percentage Error (MAPE),%	(Theilôs U)	R^2
Standard linear multiple regressions						
Model 1	0.1309	0.1113	-3.0507	107.28	0.6955	0.495
Model 1.1(w)	0.1273	0.1013	4.7343	95.0821	0.6784	0.522
Model 2	0.1285	0.1046	-0.5218	96.827	0.6870	0.513
Model 2.1(w)	0.1260	0.0997	5.4352	90.3987	0.6767	0.532
ARMA, ARMAX models						
Model 3	0.2058	0.1741	103.8938	141.4790	0.8682	-0.247
Model 4	0.1877	0.1502	94.8069	109.0351	0.9447	-0.037
Model 5	0.1291	0.1107	1.3556	104.9	0.6820	0.508
Model 6						