HIGH AND LOW PRICES AND THE RANGE IN THE EUROPEAN STOCK MARKETS : A LONG-MEMORY APPROACH

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Abstract

This paper uses fractional integration techniques to examine the stochastic behaviour of high and low stock prices in Europe and then to test for the possible existence of long-run linkages between them by looking at the range, i.e., the difference between the two logged series. Specifically, monthly, weekly and daily data on the following five European stock market indices are analysed: DAX30 (Germany), FTSE100 (UK), CAC40 (France), FTSE MIB40 (Italy) and IBEX35 (Spain). In all cases, the order of integration of the range is lower than that of the original series, which implies the existence of a long-run equilibrium relationship between high and low prices. Further, the estimated fractional differencing parameter is positive in all cases, which represents evidence of long memory.

Keywords: high and low prices, range, fractional integration

JEL Classification:

1. Introduction

can be adequately represented as a process with multiple breaks and a short-run component.

3. Methodology

When testing for cointegration in a bivariate system as in the present case the usual assumption in the literature is that the individual series are integrated of order 1, i.e., I(1), while there exists a linear combination of the two which is integrated of order 0, i.e., I(0). However, the original definition of cointegration in the seminal paper of Engle and Granger (1987) does not restrict the orders of integration to be 1 or 0, but allows for fractional values d for the original series, and an order of cointegration equal to d - b (with b > 0) for their linear combination. This is the approach followed in the present study, which allows for any real values, d and b, as the order of integration of the series of interest.

More specifically, a process $\{x_t, t = 0, \pm 1, \ldots\}$ is said to be integrated of order d, and denoted as I(d) if it can be represented as:

(1)

where L is the lag operator ($Lx_t = x_{t-1}$) and u_t is (0), defined as a covariance-stationary process with a positive and bounded spectrum. Thus, u_t can be a white noise but also a weakly autocorrela

$$-1 - \frac{-1}{2} \quad -2 \qquad .$$

Thus, if d = 0, x_t is a short-memory or I(0) process (with the effects of shocks disappearing at an exponential rate if u_t is AR(MA)), while d > 0 implies long memory behaviour, so-called because of the strong degree of dependence between observations far apart in time.[2] Note also that, if d < 0.5, x_t is covariance-stationary, while d ≥ 0.5 indicates that the series is non-stationary (in the sense that the variance of the partial sums increases in magnitude with d); further, if d < 1 the series is mean-reverting, with the effects of shocks disappearing in the long run, while d ≥1 implies lack of mean reversion, with the effects of shocks persisting forever.

In this study we analyse the relationship between high and low prices as well as the range, defined as the difference between the two logged series and therefore not estimated using a regression model. As a first step, we estimate the orders of integration of the series by using the Whittle function in the frequency domain (Dahlhaus, 1989) and following a testing procedure developed by Robinson (1994) that is suitable for statistical inference even in the case of non-stationary series. Using this method, we test the null hypothesis:

$$H_o : \; d \;\; | \;\; d_0 , \qquad (2)$$

in (1) for any real value d_0, where x_t denotes the errors in a regression model of the form:

$$y_t \;\; | \;\; \zeta \;\; 2 \;\; \eta t \;\; 2 \;\; x_t , \qquad t \;\; | \;\; 1, 2, ..., \qquad (3)$$

where y_t stands for the observed series, and α and β are unknown coefficients, specifically an intercept and a linear trend.

[2] In this case (d > 0) the shocks disappear at a hyperbolic rate.

in all cases the estimates are reported for the two cases of white noise and autocorrelated (Bloomfield) disturbances.[4]

[Insert Tables 1 and 2 about here]

In the case of the monthly series, under the assumption of white noise disturbances for both high and low prices the estimates of d are around 1 (sometimes below 1) and the unit root null hypothesis cannot be rejected in any case (see Table 1). However, those estimates are much smaller for the range, ranging between 0.27 (UK) and 0.43 (France), and the unit root null hypothesis is decisively rejected in all countries in favour of mean reversion and cointegration ($d < 1$). Interestingly, the null hypothesis $d = 0$ (consistent with the classical definition of cointegration) is also rejected this time in favour of $d > 0$. As for the results under the assumption of autocorrelated errors, the estimates of d for high and low prices are slightly smaller than the previous ones and the unit root null

Cheng et al. (2009) since they allow for the differencing parameter to take fractional values and therefore are able to capture a much greater variety of dynamic and long-run behaviours.

The empirical findings suggest that the range is mean-reverting in all cases, which implies the existence of a long-run cointegrating relationship between these two series. This confirms the well-known finding in the literature that high and low prices move together in the long run also in the case of the European stock markets and when adopting a much more general empirical framework. Further, our results indicate the presence of long-memory behaviour in both high and low prices, since the estimated value of d is always positive. This evidence of persistence goes contrary to the EMH (see Fama, 1970).

Future research could investigate whether or not the range exhibits long memory in the US case as well. Further, alternative fractional cointegration methods such as the FCVAR model proposed by Johansen and Nielsen (2010, 2012) could be used as a robustness check. Finally, the forecasting properties of the range could be examined.

References

Afzal, A. and Sibbertsen, P. (2019

Hassler, U. and Wolters, J. (1994), On the power of unit root tests against fractional alternatives. *Economic Letters*, 45: 1–5.

He, A.W. and Wan, A.T. (2009), Predicting daily highs and lows of exchange rates: a cointegration analysis. *Journal of Applied Statistics,* 36(11), 1191-1204. DOI: https://doi.org/10.1080/02664760802578304.

Johansen, S., (1991), Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica*, 59, 1551-1580.

Johansen, S. and Juselius, K., (1990), Maximum likelihood estimation and inference on cointegration – with applications to the demand for money. *Oxford Bulletin of Economics & Statistics*, 52, 169-210.

Johansen, S. and M.Ø. Nielsen (2010) Likelihood inference for a nonstationary fractional autoregressive model, *Journal of Econometrics* 158, 51-66.

Johansen, S. and M.Ø. Nielsen (2012) Likelihood inference for a Fractionally Cointegrated Vector Autoregressive Model, *Econometrica* 80(6), 2667-2732.

Parkinson, M. (1980), The extreme value method for estimating the variance of the rate of return. *Journal of Business*, 53(1), 61-65.

Robinson, P.M. (1994), Efficient Tests of Nonstationary Hypotheses, *Journal of the American Statistical Association,* 89, 1420-1437.

Taylor, M.P. and Allen, H. (1992), The use of technical analysis in the foreign exchange market. *Journal of International Money and Finance*, 11, 304-314.

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Table 1: Results with MONTHLY data and UNCORRELATED disturbances

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.96 (0.85, 1.11)	1.09 (0.95, 1.29)	1.09 (0.95, 1.29)
France	0.96 (0.85, 1.10)	1.09 (0.93, 1.32)	1.09 (0.93, 1.32)
Germany	0.95 (0.84, 1.10)	0.96 (0.83, 1.15)	0.96 (0.82, 1.15)
Italy	0.96 (0.84, 1.11)	1.05 (0.92, 1.23)	1.05 (0.92, 1.23)
U.K.	0.96 (0.84, 1.10)	1.01 (0.87, 1.20)	1.01 (0.87, 1.20)
Series: LOW	No terms	An intercept	A linear trend
Spain	0.96 (0.86, 1.11)	1.05 (0.90, 1.26)	1.05 (0.90, 1.26)
France	0.96 (0.85, 1.11)	0.93 (0.78, 1.14)	0.93 (0.78, 1.14)

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Table 3: Results with WEEKLY data and UNCORRELATED disturbances

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.99 (0.93, 1.05)	1.11 (1.04, 1.19)	1.11 (1.04, 1.19)
France	0.99 (0.93, 1.05)	1.07 (1.00, 1.15)	1.07 (1.00, 1.15)
Germany	0.99 (0.93, 1.05)	1.07 (1.00, 1.14)	1.06 (1.00, 1.14)
Italy	0.99 (0.93, 1.06)	1.16 (1.09, 1.25)	1.16 (1.09, 1.25)
U.K.	0.99 (0.93, 1.05)	1.06 (0.99, 1.15)	1.06 (0.99, 1.15)
Series: LOW	No terms		

Table 4: Results with WEEKLY data and AUTOCORRELATED disturbances

Series: HIGH	No terms	An intercept	A linear trend

Table 6: Results with DAILY data and AUTOCORRELATED disturbances

Series: HIGH	No terms	An intercept	A linear trend
Spain	0.99 (0.95, 1.04)	0.94 (0.89, 1.00)	0.94 (0.89, 1.00)
France	1.00 (0.95, 1.05)	0.93 (0.88, 0.99)	0.93 (0.88, 0.99)
Germany	0.99 (0.95, 1.05)	0.97 (0.92, 1.03)	0.97 (0.92, 1.03)
Italy	1.00 (0.95, 1.06)	0.96 (0.91, 1.01)	0.96 (0.91, 1.01)
U.K.	1.00 (0.95, 1.06)	0.91 (0.86, 0.99)	0.91 (0.86, 0.99)
Series: LOW	No terms	An intercept	A linear trend
Spain	1.00 (0.95, 1.05)	0.89 (0.85, 0.94)	0.89 (0.85, 0.94)
France	1.00 (0.95, 1.06)	0.87 (0.83, 0.92)	0.87 (0.83, 0.92)
Germany	1.00 (0.95, 1.06)	0.91 (0.87, 0.96)	0.91 (0.87, 0.96)
Italy	1.00 (0.95, 1.05)	0.90 (0.86, 0.95)	0.90 (0.86, 0.95)
U.K.	1.00 (0.95, 1.05)	0.90 (0.86, 0.94)	0.90 (0.86, 0.94)
Series: RANGE	No terms	An intercept	A linear trend
Spain	0.44 (0.40, 0.48)	0.39 (0.34, 0.43)	0.37 (0.33, 0.42)
France	0.45 (0.41, 0.49)	0.41 (0.37, 0.46)	0.40 (0.36, 0.45)
Germany	0.44 (0.41, 0.48)	0.41 (0.37, 0.45)	0.41 (0.37, 0.45)
Italy	0.46 (0.41, 0.50)	0.42 (0.37, 0.45)	0.41 (0.36, 0.46)
U.K.	0.49 (0.45, 0.53)	0.45 (0.41, 0.50)	0.45 (0.41, 0.50)

See the Notes to Table 1.

Table 8: Summary of the results for the weekly series

Country	Series	No autocorrelation	Autocorrelation
Spain	High	1.11 (1.04, 1.19)	0.98 (0.89, 1.09)
	Low	1.03 (0.96, 1.12)	0.84 (0.75, 0.94)
	Range	0.34 (0.28, 0.41)	0.30 (0.20, 0.41)
France	High	1.07 (1.00, 1.15)	0.94 (0.84, 1.07)
	Low	0.99 (0.92, 1.08)	0.78 (0.69, 0.89)
	Range	0.40 (0.34, 0.48)	0.36 (0.27, 0.48)
Germany	High	1.07 (1.00, 1.14)	1.00 (0.88, 1.14)
	Low	1.05 (0.97, 1.14)	0.80 (0.71, 0.91)
	Range	0.41 (0.36, 0.47)	0.43 (0.34, 0.54)
Italy	High	1.16 (1.09, 1.25)	0.98 (0.89, 1.11)
	Low	1.06 (0.99, 1.15)	0.84 (0.75, 0.96)
	Range	0.42 (0.35, 0.49)	0.31 (0.22, 0.42)
UK	High	1.06 (0.99, 1.15)	0.91 (0.81, 1.02)
	Low	1.02 (0.94, 1.11)	0.76 (0.68, 0.87)
	Range	0.40 (0.34, 0.47)	0.35 (0.27, 0.47)

See the Notes to Table 1.

Table 9: Summary of the results for the daily series

Country	Series	No autocorrelation	Autocorrelation
Spain	High	1.06 (1.02, 1.10)	0.94 (0.89, 1.00)
	Low	1.06 (1.02, 1.10)	0.89 (0.85, 0.94)
	Range	0.34 (0.31, 0.37)	0.37 (0.33, 0.42)
France	High	1.04 (1.00, 1.08)	0.93 (0.88, 0.99)
	Low	1.05 (1.01, 1.09)	0.87 (0.83, 0.92)
	Range	0.37 (0.34, 0.40)	0.41 (0.37, 0.46)
Germany	High	1.05 (1.02, 1.09)	0.97 (0.92, 1.03)
	Low	1.06 (1.02, 1.10)	0.91 (0.87, 0.96)
	Range	0.36 (0.33, 0.38)	0.41 (0.37, 0.45)
Italy	High	1.07 (1.03, 1.11)	0.96 (0.91, 1.01)
	Low	1.08 (1.04, 1.13)	0.90 (0.86, 0.95)
	Range	0.36 (0.34, 0.39)	0.42 (0.37, 0.45)
UK	High	1.09 (1.05, 1.14)	0.91 (0.86, 0.99)
	Low	1.06 (1.02, 1.11)	0.90 (0.86, 0.94)
	Range	0.34 (0.32, 0.37)	0.45 (0.41, 0.50)

See the Notes to Table 1.