Department of
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LINKAGES BETWEEN

1. Introduction

Globalisation has led to international financial markets becoming increasingly interconnected, with equities displaying a high degree of co-movement across countries. This paper analyses linkages between US and European stock markets. Specifically, it applies fractional integration and cointegration techniques with the aim of testing co-movement between the S&P 500 Index and the Euro Stoxx 50 Index over the period from 1986 t

2. Literature review

There is an extensive literature testing whether stock prices are unpredictable and follow a random walk as implied by the Efficient Market Hypothesis or are instead mean-the returns as a proxy for volatility, and estimated a long-memory process to examine persistence in volatility, establishing some stylized facts regarding the temporal and distributional properties of these series. However, in a following study, Granger and Ding (1996) found that the parameters of the long memory model varied considerably across subsamples. The issue of fractional integration with structural breaks in stock markets has been examined by Mikosch and Starica (2000) and Granger and Hyung (2004) among others. Stochastic volatility models using fractional integration have been estimated by Crato and de Lima (1994), Bollerslev and Mikkelsen (1996), Ding and Granger (1996), Breidt, Crato and de Lima (1997, 1998), Arteche (2004), Baillie, Han, Myers and Song (2007), etc.

Another strand of the literature focuses not only on individual time series, but also on the co-movement between international stock markets. It dates back to Panto et al. (1976), who used correlations to test for stock market interdependence. Subsequent studies relied on the cointegration framework developed by Engle and Granger (1987) and Johansen (1991, 1996) to examine long-run linkages. For instance, Taylor and Tonks (1989) showed that markets in the US, Germany, Netherlands and Japan exhibited cointegration over the period October 1979 - June 1986. Jeon and Von-Furstenberg (1990) used the VAR approach and found an increase in cross-border cointegration since 1987. For post-crash periods and times of heightened volatility, Lee and Kim (1994) showed that the US and Japanese markets had tighter linkages. Copeland and Copeland (1998) and Jeong (1999) found a leadership role for the US relative to smaller markets. Wong et al. (2005) used fractional cointegration and reported linkages between India and the US, the UK and Japan. Syllignakis and Kouretas (2010) studied instead the integration of European and US stock markets, finding strong long-run linkages between US and German stock prices. Bastos and

Caiado (2010) found evidence of cointegration for a wider sample of forty-six developed and emerging countries. The present study contributes to this literature by using fractional cointegration techniques to test for long-run linkages between the US and European financial markets and highlighting a change in their relationship.

3. Empirical methodology

The empirical analysis is based on the concepts of fractional integration and cointegration. For our purposes, we define an I(0) process as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. Therefore, a time series $\{x_t, t = 1, 2, \ldots\}$ is said to be I(d) if it can be represented as

$$(1 - L)^d x_t = u_t, \quad t = 0, ^\circ 1, \ldots, \qquad (1)$$

with $x_t = 0$ for $t \leq 0$, where L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is $I(0)$. By allowing d to be fractional, we introduce a much higher degree of flexibility in the dynamic specification of the series in comparison to the classical approaches based on integer differentiation, i.e., d = 0 and d = 1.

Given the parameterisation in (1), different models can be obtained depending on the value of d. Thus, if d = 0, $x_t = u_t$, x_t is said to be "*short memory*", and the observations may be weakly autocorrelated, i.e. with the autocorrelation coefficients decaying at an exponential rate; if d > 0, x_t is said to be "*long memory*", so named because of the strong association between observations far apart in time. If d belongs to the interval (0, 0.5) x_t is still covariance stationary, while d ≥ 0.5 implies nonstationarity. Finally, if d < 1, the series is mean reverting, implying that the effect of the shocks disappears in the long run, in contrast to what happens if d ≥ 1, when the effects of shocks persist forever.

We estimate d using a parametric Whittle function in the frequency domain (Fox and Taqqu, 1986; Dahlhaus, 1989) along with a Lagrange Multiplier (LM) test developed by Robinson (1994a) that has the advantage that it remains valid even in the presence of nonstationarity. Some semi-parametric methods (Robinson, 1995a,b) will also be used for the analysis.

For the multivariate case, we apply fractional cointegration methods. First we test for homogeneity in the orders of integration of the two series using a procedure developed by Robinson and Yajima (2002); then, since the two parent series appear to be I(1), we run a standard OLS regression of one variable against the other, and examine the order of integration of the estimated errors. A Hausman test of the null hypothesis of no cointegration against the alternative of fractional cointegration (Marinucci and Robinson, 2001) is also carried out.

4. Data and empirical results

The series used for the analysis are the S&P 500 Index and the Euro Stoxx 50 Index (downloaded from Yahoo! Finance)

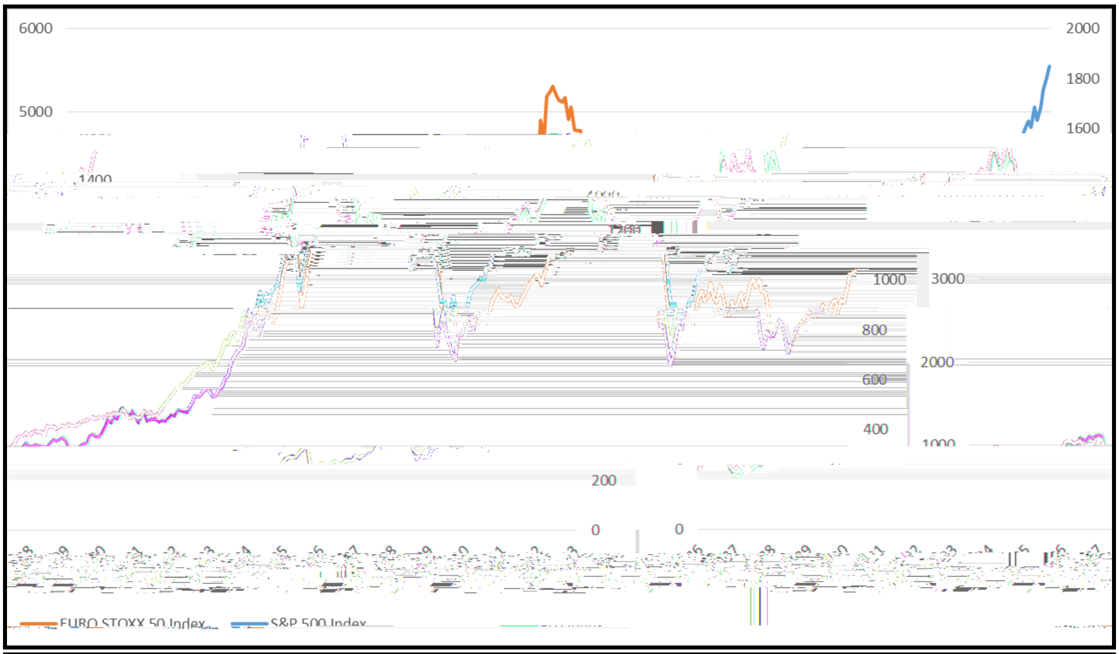

Figure 1 displays the two series. They exhibit very similar behaviour from the beginning of the sample until 2009, with two peaks occurring in 2000 and 2007, followed by a sharp decline in 2001 and 2008. Since the start of the recovery from the global financial crisis in 2009, a much faster recovery is observed in the S&P 500 than in Euro Stoxx 50.

As a first step we estimate the fractional differencing parameter in the following model,

$$y_t = b_0 + b_1 t + x_t, \quad (1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots \tag{2}$$

where y_t is the observed series, β_0 and β_1 are the coefficients corresponding to an intercept and a linear time trend, and x_t is assumed to be I(d), where d can take any real value. Therefore the error term, u_t, is I(0), and is assumed 61.18 T 0 0 1 250.49 161.18 Tm /P <8ia> BDC

Table 1 shows the estimates of the fractional differencing parameter for the original series, while Table 2 focuses on the log-transformed data. In both cases, we display the estimates of d, along with their corresponding 95% confidence intervals, in the three cases of no regressors ($\beta_0 = \beta_1 = 0$ a priori in (2)), an intercept (β_0 unknown and $\beta_1 = 0$ a priori) and an intercept with a linear trend (β_0 and β_1 unknown).

For the original series (see Table 1), if u_t

roots in stock indices in most developed economies (Huber, 1997; Liu et al., 1997; Ozdemir, 2008; Narayan, 2005, 2006; Narayan and Smyth, 2004, 2005; Qian et al., 2008; etc.).

Various studies in the literature have documented non-linear dynamics in stock prices. For instance, Hsieh (1991) explored 'Chaos Dynamics' in stock prices not following a normal distribution; Abhyankar et al. (1995) provided evidence of non-linearity in the London Financial Times Stock Exchange (FTSE) index that cannot be fully explained by a GARCH model; Kosfeld and Robé (2001) showed various types of non

the full sample analysis; since both series appear to be I(1) throughout the sample it is legitimate to test for cointegration.

A necessary condition for cointegration is that the two parent series have the same degree of integration. In our case, the confidence intervals reported in Table 1 and 2 clearly suggest that the unit root (I(1)) hypothesis cannot be rejected for either series. However, we also perform a test of the homogeneity of the orders of integration in the bivariate systems (i.e., H_o: $d_x = d_y$), where d_x and d_y are the orders of integration of the two individual series, by using an adaptation of the Robinson and Yajima (2002) statistic $\hat{T}_{xy}$ to log-periodogram estimation. This is calculated as:

$$\hat{T}_{xy} = \frac{m^{1/2}\left(\hat{d}_x - \hat{d}_y\right)}{\left(\frac{1}{2}\left(1 - \hat{G}_{xy} / (\hat{G}_{xx}\hat{G}_{yy})\right)\right)^{1/2} + h(n)} \qquad (5)$$

where h(n) > 0 and $\hat{G}_{xy}$ is the (xy)th element of

$$\hat{G} = \frac{1}{m}\sum_{j=1}^{m} \text{Re}\left[\hat{\Lambda}(\lambda_j)^{-1} I(\lambda_j) \hat{\Lambda}(\lambda_j)^{-1*}\right], \quad \hat{\Lambda}(\lambda_j) = diag\left\{ e^{i\pi \hat{d}_x/2} \lambda^{-\hat{d}_x}, e^{i\pi \hat{d}_y/2} \lambda^{-\hat{d}_y} \right\},$$

with a standard normal limit distribution (see Gil-Alana and Hualde (2009) for evidence on the finite sample performance of this procedure). As expected, the results strongly support the hypothesis that the two orders of integration are the same, with a unit root being present in both cases.

Next, we examine the cointegrating relationship by estimating the following regression,

$$y_{1t} = \beta_0 + \beta_1 y_{2t} + x_t, \qquad (1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots \qquad (6)$$

where y_{1t} is the logged S&P 500 Index and y_{2t} the logged Euro Stoxx 50 Index. We consider the two cases of uncorrelated (white noise) and correlated (Bloomfield) errors.

prices. In the case of the latter, other factors such as less prominence of technology stocks have also resulted in underperforming stock indices.

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Table 1: Estimates of d for each series using the raw data

i) White noise disturbances			
	No regressors	An intercept	A linear time trend
U.S. stock market	1.09 (1.02, 1.17)	1.07 (1.00, 1.15)	1.07 (1.00, 1.15)
Euro stock market	1.10 (1.03, 1.18)	1.10 (1.04, 1.19)	1.10 (1.09, 1.19)
ii) AR(1) disturbances			
U.S. stock market	1.09 (0.98, 1.22)	1.05 (0.92, 1.17)	1.05 (0.93, 1.17)
Euro stock market	1.10 (0.96, 1.24)	1.09 (0.97, 1.23)co	1.09 (0.97, 1.23)
iii) Bloomfield disturbances			
U.S. stock market	1.08 (0.96, 1.21)	**1.04 (0.93, 1.19)**	1.04 (0.93, 1.19)
Euro stock market	1.11 (0.98, 1.25)	**1.09 (0.98, 1.23)**	1.09 (0.98, 1.23)
iv) monthly AR(1) disturbances			
U.S. stock market	1.08 (1.02, 1.17)	1.06 (0.99, 1.15)	1.06 (0.99, 1.15)
Euro stock market	1.09 (1.03, 1.18)	1.10 (1.03, 1.19)	1.10 (1.03, 1.19)

Table 5

Figure 1: Time series plots: US and European stock market indices

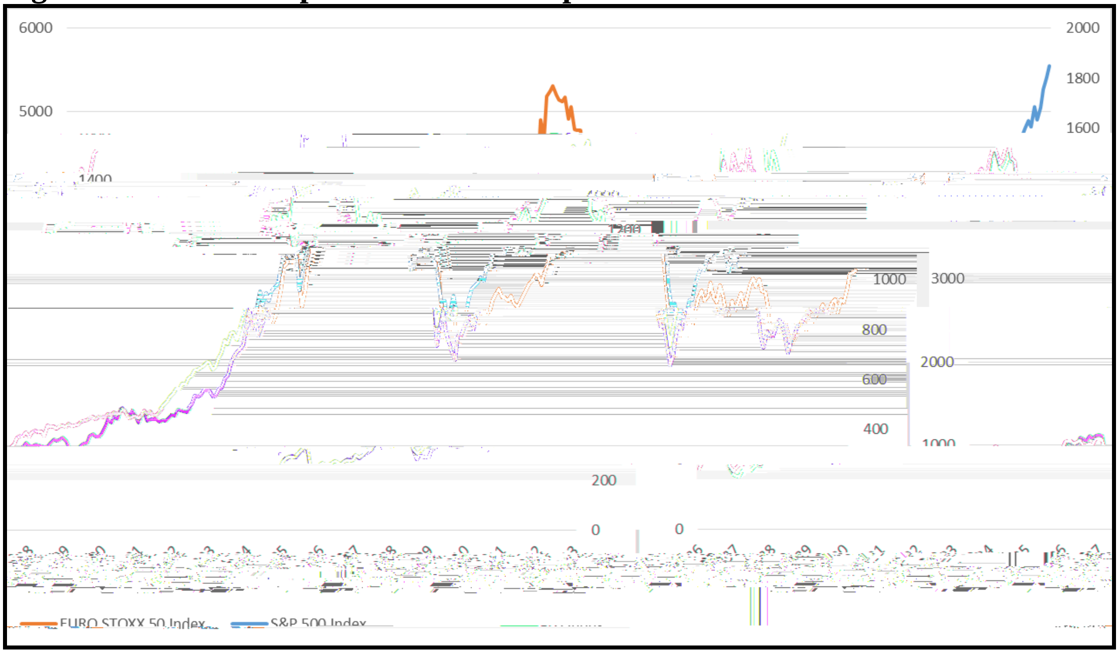

Figure 3: Recursive estimates of d from the cointegrating regression

The thick line refers to the estimated values of d. The thin lines are the 95% confidence intervals.