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**A Non-linear Analysis of Gibson's Paradox  
in the Netherlands, 1800-2012**

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**A NON-LINEAR ANALYSIS OF GIBSON'S PARADOX  
IN THE NETHERLANDS, 1800-2012**

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**Abstract**

This paper adopts a multivariate, non-linear framework to analyse Gibson's paradox in the Netherlands over the period 1800-2012. Specifically, SSA (singular spectrum) and MSSA (multichannel singular spectrum) techniques are used. It is shown that changes in monetary policy regimes or volatility in the price of gold by themselves cannot account for the behaviour of government bond yields and prices in the Netherlands over the last 200 years. However, the inclusion of changes in the real rate of return on capital, M1, primary credit rate, expected inflation, and money purchasing power enables a nonlinear model to account for a sizeable percentage of the total variance of Dutch bond yields.

**Keywords:** *Gibson's paradox, (Multichannel) Singular spectrum analysis, Interest rates, Causality, The Netherlands*

**JEL Classification:** *E50, E4, C39, C53*

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## **1. INTRODUCTION**

The long-run relationship between bond yields and prices, first noticed by Gibson (1923), has become known in the literature as Gibson's paradox. Numerous studies have been carried out and have failed to account for it: Friedman and Schwartz (1976) in fact defined it as an "empirical phenomenon without a theoretical explanation". The original study by Gibson (1923) examined the long-run correlation between the cost of living and bond yields (consols) in Britain. Fisher (1930) focused on expected inflation to explain the difference between nominal and real interest rates and

## **2. DATA AND METHODOLOGY**

### **2.1 Data sources and description**

This study uses a data set including 73 variables that could be relevant to explaining Gibson's paradox in the Netherlands over the period 1800-2012. The series have been obtained from the following sources: Zanden and Riel (2000), Zanden and Lindblad (1989), Smits et al. (2000), Hart et al. (2010), van Bochove and Huitker (1987), Netherland (2010), (Netherland, 2012), CBS - Historical series - Publications (2013), International Institute of Social History (2013), Homer and Sylla (2011), Jacobs and Smits (2001), Barro and Ursúa, (2008), Officer and Williamson (2013), Bos (2008), Groote et al. (1996), Homepage Statistics - De Nederlandsche Bank (2013), van den End (2011).

In some cases observations were missing for the periods 1800-1813, 1913-1917, 1939-1946. In such cases the SSA Reconstruction/Prediction filter (Kspectra program 3.4) has been used to fill the gaps, following Harvey (1990), Hamilton (1994), and Priestley (1981). Samples and variable descriptions are listed in Table A1 in the Appendix. In the end only 15 of the 73 variables were included in the empirical model, on the basis of data availability, theoretical considerations and pre-processing analysis (PCA screening).

### **2.2 Descriptive analysis**

Co-movement between long-term interest rates (LR - yields on Consols) and consumer prices (CPI) as described in Gibson (1923) is clearly present in the Netherlands (see Figure 1). However, a break occurred after WWII. An even stronger correlation can be seen between short-term rates and prices (see Figure 2).

**Insert Figure 1 around here**

**Insert Figure 2 around here**

Further evidence is provided by Table 1, which reports the correlation coefficients for different sub-samples. It would suggest that Gibson's paradox only existed in the 19th



(with the correlation between LR and CPI b

The Phase spectrum shows the lead/lag relationship between LR and LCPI and can be expressed as in Warner (1998):

$$\phi_{x,y}(\omega) = \arctan \frac{\text{Im } g_{x,y}(\omega)}{\text{Re } g_{x,y}(\omega)} . \quad (3)$$

At high frequencies, when the squared coherency is largest, the phase spectrum has positive values, suggesting that price changes lead to LR changes.

Non-linearity tests (not reported here), specifically the BDS test (see Broock et al. (1996)) and one of its variants (see Koenda (2001)) detect nonlinear behaviour in the LR, CPI and LCPI series. For this reason, SSA (singular spectral analysis) and MSSA (multichannel singular spectral analysis) techniques are used here. SSA detects trends, oscillatory patterns and noise in the series. The single channel SSA involves two main steps: decomposition and reconstruction. Decomposition is obtained by embedding the original time series (LR, LCPI) into lagged vector sequences of the form (trajectory matrix) (see Golyandina and Zhigljavsky (2013)):

$$X = \begin{matrix} & & & f_1 & f_2 & \dots & f_K & & & f_L \\ & & & f_2 & f_3 & \dots & f_{L-1} & & & \\ & & & M & M & \dots & O & & & M \\ & & & f_K & f_{K-1} & \dots & L & & & f_n \end{matrix} . \quad (4)$$

A singular value decomposition of the trajectory matrix (4) has the form  $X = X_1 + \dots + X_d$  with  $X_i = \sqrt{\lambda_i} U_i V_i^T$ . The diagonal averaging method is applied to reconstruct the original LR time series as the sum of the identified principal components. The Eigentriple grouping takes the form  $X = X_{l_1} \dots X_{l_m}$  with the input series LR decomposed as

$$x_n = \sum_{k=1} \tilde{x}_n^{(k)}$$





(10)

st error from the



after two years, suggesting long memory in both series. The coherence is above 0.8 for short periods (higher frequencies), then drops before reverting back to 0.8 in the long run. The Coherency function indicates that LR and LCPI are highly correlated both in the short and long run.

***Insert Figure 5 around here***

The Gain factor, equivalent to the regression coefficient in the time domain, is higher for the log of the price level (LCPI) than for LR (see Figure 6). The high coherence implies that changes in LR transmitted to LCPI are magnified 1-2 times, whilst in the opposite direction the corresponding percentage is only 20-100%.

**Insert Figure 6 around here**

The results from spectral analysis can be summarised as follows:

1. The coherence between LR and LCPI is high, implying that movements in one series have strong and long-lasting effects on the other, with LCPI affecting LR in particular.
2. The gain coefficient is higher for LCPI than for LR, indicating that movements in the former have a larger impact on the latter than vice versa.
3. The phase angle is close to zero and  $2\pi$ , therefore LR and LCPI are simultaneously affecting each other both in the short and long run.

Overall, the spectral analysis provides evidence of strong correlation between LR and LCPI (i.e., of a Gibson's paradox in the Netherlands), but gives no information on causality linkages. A Multichannel Singular Spectrum Causality (MSSA) test, outlined in Section 3.3, is therefore carried out to shed light on causation.

### **3.2 Multichannel Singular Spectrum Analysis**

On the basis of the Cochrane-Orcutt AR(1) procedure and logit regression results (not reported here) we select 15 (statistically most significant) variables from the pool of 73 examined. The selected variables are CPI, DEBT, VOL, EXPSTIR, GOLDSIL, TURNOVER, RORK2, UN, M1, EXPP, DR, NIR6, EXP3, PPG (see table A1). VOL stands for gold price volatility and is a proxy for the gold standard regime shift, which had no effect on the price-interest rate link. DR is the DNB bank discount rate (primary credit rate) and is a proxy for inflation policy regime shifts, which appear to have a significant impact instead. Overall, the MSSA model confirms that Gibson's paradox was still present in the Netherlands after the WWII, in fact until

been found, the null hypothesis of transformed LR series following an AR(1) red noise process cannot be rejected. Thus, movements in LR cannot be explained by trend, periodic or oscillatory components in the series. This implies that the MSSA analysis of the bivariate relationship between LR and LCPI is necessary to describe the LR dynamics.

### **Insert Figure 8 around here**

Figure 8 shows the statistically significant oscillatory patterns in LR and LCPI around the frequency 0.18-0.20, which indicates a possible five-year cycle. Since they do not repeat themselves over time, they cannot be defined as cycles in Gibson's paradox. The calculated eigenvalue pairs are not statistically significant for either short or high frequencies. This implies that changes in LR and LCPI during the 1-4 year period and  $> 6$  periods have no significant impact on the relationship between LR and LCPI. Variations in Gibson's paradox can be explained by fluctuations in interest rates and prices over the 5-5.5 year's period. Changes in the cost of living have no direct and immediate effect on long-term interest rates. This is in accordance with the results of Mises and Greaves (2011). The eigenvalues functions for LR and LCPI (not displayed here) in fact slowly increase from 0, reaching a maximum after 5.5 years and decaying slowly afterwards.

The MSSA analysis once again supports the existence of Gibson's paradox in the Netherlands, identifying 5-5.5 year oscillations in LR and LCPI. The results are presented in Table 2, which displays the identified principal (spectral) components and the associated total variance explained by the significant eigenvalues pairs.

Table 2

*Multivariate SSA: variable-based results (m=50)*

Series	Frequency	Power	% of variance Explained	% of cumulative variance Explained
LR*	0.003	61.33	9.18	9.18
LCPI	0.018	16.51	2.47	11.65
DEBT	0.035	13.63	2.04	13.69
VOL	0.082	13.52	2.02	15.71
EXPSTIR*	0.111	13.46	2.01	17.72
GOLDSIL	0.03	13.01	1.95	19.67
TURNOVER	0.081	12.96	1.94	21.61
RORK2*	0.111	12.02	1.8	22.69
UN	0.019	11.68	1.75	24.44
M1*	0.174	11.22	1.68	26.12
EXPP*	0.174	10.77	1.61	27.73
DR*	0.202	10.59	1.58	29.31
NIR6*	0.202	10.53	1.58	30.89
EXP3*	0.136	10.39	1.56	32.45
PPG*	0.136	10.05	1.50	33.95

Source: Author's calculations

Notes. \* patterns significantly ( $p < .05$ ) different from the Monte Carlo (1000) simulated red noise

Table 2 confirms the multivariate and complex nature of Gibson's paradox (see also Figure 9). Of the 15 spectral components, 4 eigenvalue pairs passed the Monte Carlo test. The price level (LCPI) and unemployment (UN) eigenvalue pair is only weakly significant, with eigenvalues towards the upper bounds of the confidence interval. Autoregressive (long

The pairs EXPSTIR and RORK2 account for 3.81% of the total variance of the series. Short-term interest rates and returns on capital are clearly distinguishable from the AR(1) red noise process. The DEBT and GOLDSIL eigenvalue pair instead fail to pass a red noise Monte Carlo test (i.e., they are not significantly different from the red noise), and the same holds for the VOL and TURNOVER eigenvalue pair. The other pairs (M1 and EXPP; DR and NIR6; EXP3 and PPG) capture 3.29%, 3.16% and 3.06% respectively of the total variance and in all cases are significantly different from the simulated red noise at the 95% confidence level.

**Insert Figure 9 around here**

LCPI and UN are not statistically significant, which suggests that some important oscillatory components might have been left out; therefore, an MSSA model with a different windows size ( $m=100$ ) is also estimated following Golyandina et al. (2010) and the general rule  $m=N/2$ . As expected, choosing a bigger window size results in more significant oscillatory patterns being identified (see Figure 10).

**Insert Figure 10 around here**

Both the LR and LCPI oscillations now appear to be significantly different from the simulated red noise process. VOL and EXPSTIR are now weakly significant, as are the other previously identified significant eigenvalues pairs (implying rejections of the red noise null hypothesis), as can be seen from Table 3.

Estimating MSSA with different window sizes leads to the same conclusion. Overall, there is clear evidence that Gibson's paradox in the Netherlands is a multivariate phenomenon that can only be explained by a large number of variables. Nine (9) PCA

components (statistically significant eigenvalues) were identified with an MSSA ( $m=100$ ) model explaining 34.58% of the total variance.

Table 3

*Multivariate SSA: variable-based results ( $m=100$ )*

Series	Frequency	Power	% of variance explained	% of cumulative variance explained
LR*	0.000	74.53	6.03	6.03
LCPI*	0.003	52.91	4.28	10.31
DEBT	0.02	27.66	2.24	12.55
VOL	0.081	27.25	2.21	14.76
EXPSTIR	0.082	27.15	2.20	16.96
GOLDSIL	0.031	25.02	2.02	18.98
TURNOVER	0.032	24.64	1.99	20.97
RORK2*	0.111	24.12	1.95	22.92
UN	0.021	23.82	1.93	24.85
M1*	0.125	20.77	1.68	26.53
EXPP*	0.176	20.31	1.64	28.17
DR*	0.176	20.19	1.63	29.8
NIR6*	0.202	19.73	1.60	31.4
EXP3*	0.109	19.67	1.59	32.99
PPG*	0.203	19.65	1.59	34.58

Source: Author's calculations

Notes. \* patterns significantly ( $p < .05$ ) different from the Monte Carlo (1000) simulated red noise

The patterns emerging from the table above were reconstructed in the time domain using MSSA for tracing a limit cycle. The identified cycle has a period of 5-5.5 years and





bidirectional feedback is found in four cases, namely between LR and LCPI, LR and M1, LR and DR, LR and EXP3.

Table 4

*Multivariate Spectral Granger Causality Analysis*

<i>Causality relation</i>	$F_{X Y}^{(h,d)}$	$F_{Y X}^{(h,d)}$
<i>MSSA forecast MSE of LR (LCPI as second series)</i>	<i>(LCPI LR) 0.99*</i>	<i>(LR LCPI) 0.81*</i>
<i>MSSA forecast MSE of LR (RORK2 as second series)</i>	<i>(RORK2 LR) 0.99*</i>	<i>(LR RORK2) 1.15</i>
<i>MSSA forecast MSE of LR (M1 as second series)</i>	<i>(M1 LR) 0.99*</i>	<i>(LR M1) 0.29*</i>
<i>MSSA forecast MSE of LR (EXPP as second series)</i>	<i>(EXPP LR) 1.00</i>	<i>(LR EXPP) 0.80*</i>
<i>MSSA forecast MSE of LR (DR as second series)</i>	<i>(DR LR) 0.95*</i>	<i>(LR DR) 0.98*</i>
<i>MSSA forecast MSE of LR (NIR6 as second series)</i>	<i>(NIR6 LR) 1.01</i>	<i>(LR NIR6) 0.18*</i>
<i>MSSA forecast MSE of LR (EXP3 as second series)</i>	<i>(EXP3 LR) 0.98*</i>	<i>(LR EXP3) 0.89*</i>
<i>MSSA forecast MSE of LR (PPG as second series)</i>	<i>(PPG LR) 0.98*</i>	<i>(LR PPG) 1.70</i>

Source: Author's calculations

Notes. (X → Y) X Granger cause Y and (Y → X) Y Granger cause X

Table 5

*Diebold-Mariano Test Statistics for h Step Ahead Forecast (h=40)*

<i>Forecasting Method</i>	<i>Test Statistics</i>	<i>P-value</i>
SSA of LR	1.424	-
SSA of LR against MSSA of LR (LCPI as second series)	1.423*	0.92
SSA of LR against MSSA of LR (RORK2 as second series)	1.416*	0.47
SSA of LR against MSSA of LR (M1 as second series)	1.442	0.53
SSA of LR against MSSA of LR (EXPP as second series)	1.438	0.36
SSA of LR against MSSA of LR (DR as second series)	1.404*	0.05
SSA of LR against MSSA of LR (NIR6 as second series)	1.457	0.009
SSA of LR against MSSA of LR (EXP3 as second series)	1.403*	0.27
SSA of LR against MSSA of LR (PPG as second series)	1.406*	0.22

Source: Author's calculations

Notes. The null hypothesis is that the forecast accuracy is the same

The MSSA forecasting method provides better forecasts; \* indicates higher forecast accuracy

The test results show that the MSSA-based forecasts outperform the SSA-based ones (lower MSE), supporting the conclusions from the multivariate spectral Granger causality analysis.

#### 4. CONCLUSIONS

This paper investigates Gibson's paradox in the Netherlands over the period 1800-2012 using nonlinear spectral methods (namely, SSA and MSSA) and a large set of potential explanatory variables, unlike previous studies considering at most two. The fifteen variables included in the selected model explain 35% of the total variance in bond yields over the period examined, providing strong evidence of the existence of the paradox in the Netherlands, as already documented by Fields (1984), Fase (1972) (whilst the opposite conclusion was reached by Ram (1987)). An even higher percentage of the variance could have been explained if

variables such as the DNB gold reserves could have been included, but unfortunately the relevant data are only available from 1875. Although policy regime changes (such as the adoption of the classical gold standard or the loss of central bank independence when the Netherlands joined EMU) appear to affect the relationship, as also argued by Cogley et al. (2011) in the case of the US), they cannot account for it since it appears to be present under all the various regimes.

The empirical results indicate the existence of a 5-5.5 year oscillatory pattern in long-term bond yields in the Netherlands. They also suggest that Gibson's paradox is a very complex phenomenon, that cannot easily be explained by exogenous shocks (Keynes' effect, Fishers' effect, Summers' effect, Wicksell's effect) or standard liquidity preference, loanable funds and rational expectations models. Non-linearities are clearly an important feature of this relationship which should be modelled explicitly to understand its dynamic properties.

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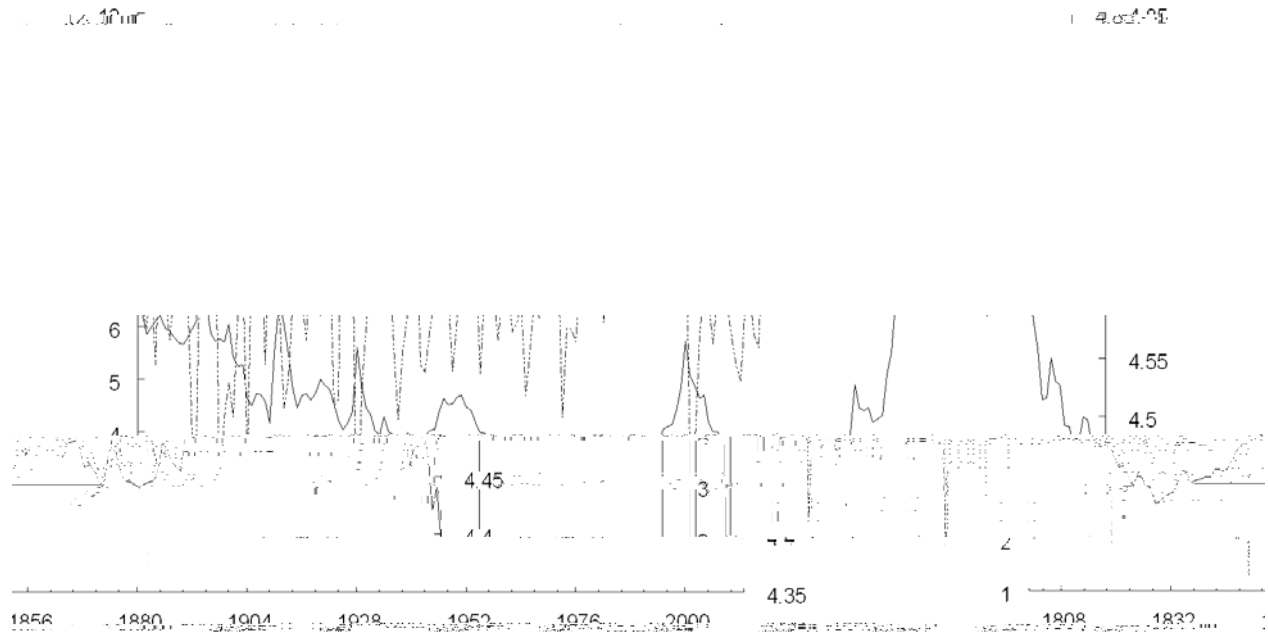
## Appendix

Table A1 *Time Series Data List and Model Variables Description*

Time Series ID	Sample	Variable description
LR	1800-2012	Long-term interest rate yield on Consols (government bonds) in %

## List of Figures

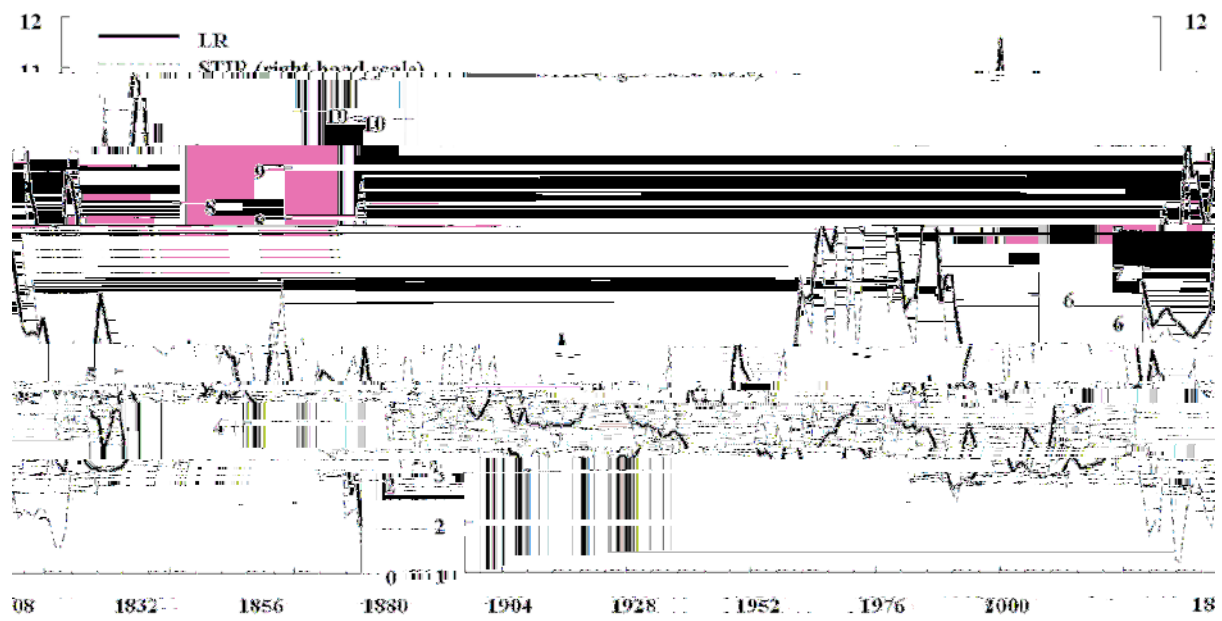
## Figure 1

*Price Level and Bond Yields Dynamics in the Netherlands 1800-2012*

Source: Authors' calculations

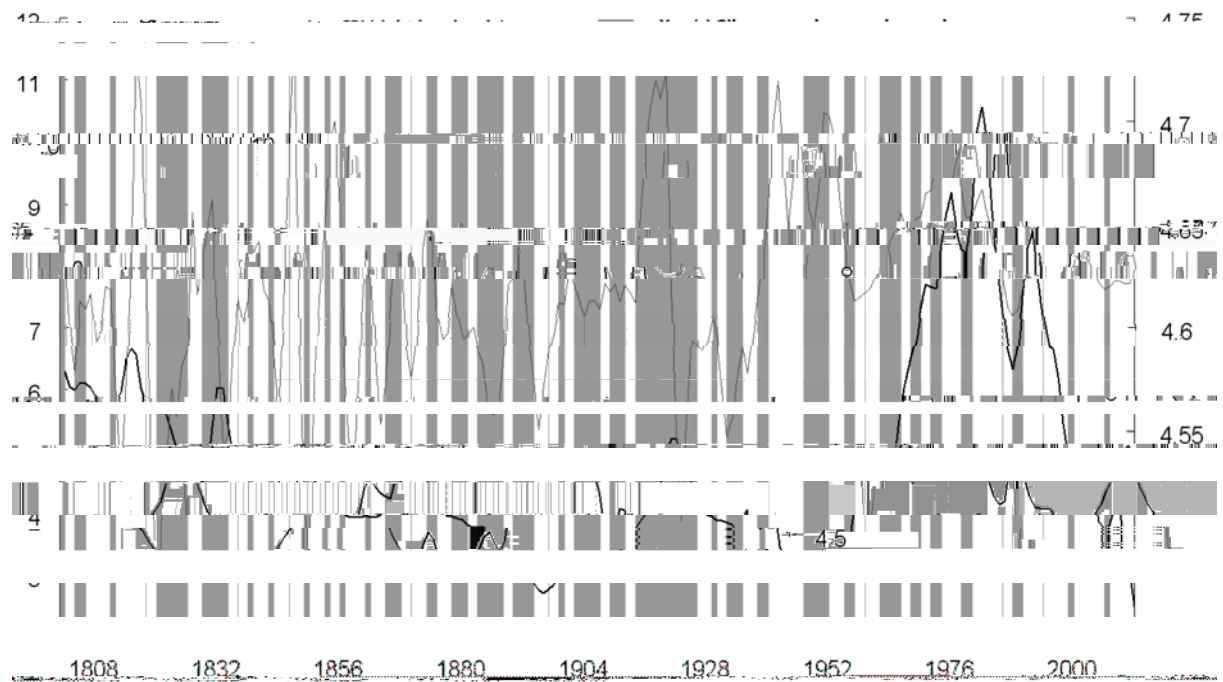


Figure 2

*Bond Yields and Short Term Interest Rates Dynamics in the Netherlands 1800-2012*

Source: Authors' calculations

Figure 3

*Rolling Window Correlation Between LR and LCPI in the Netherlands 1800-2012*

Source: Authors' calculations

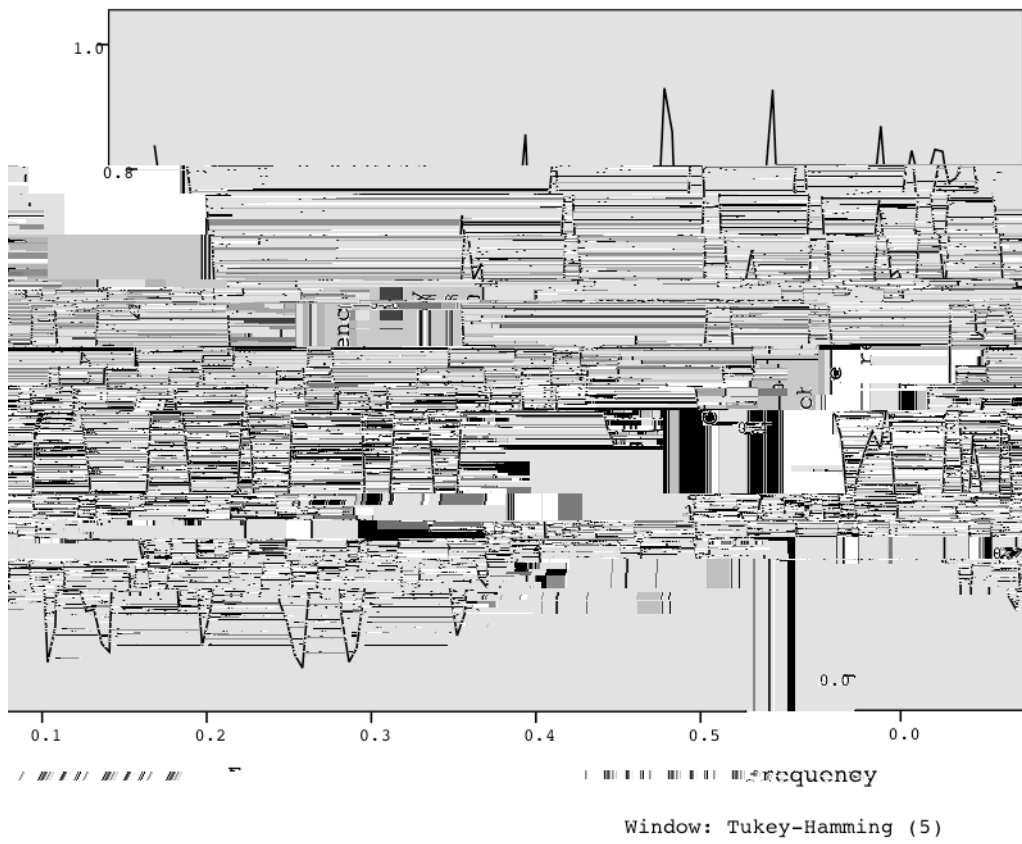
Notes: Three Year Moving average curves for LR and LogCPI

Figure 4

*Phase Spectrum of CPI and LR by Frequency*

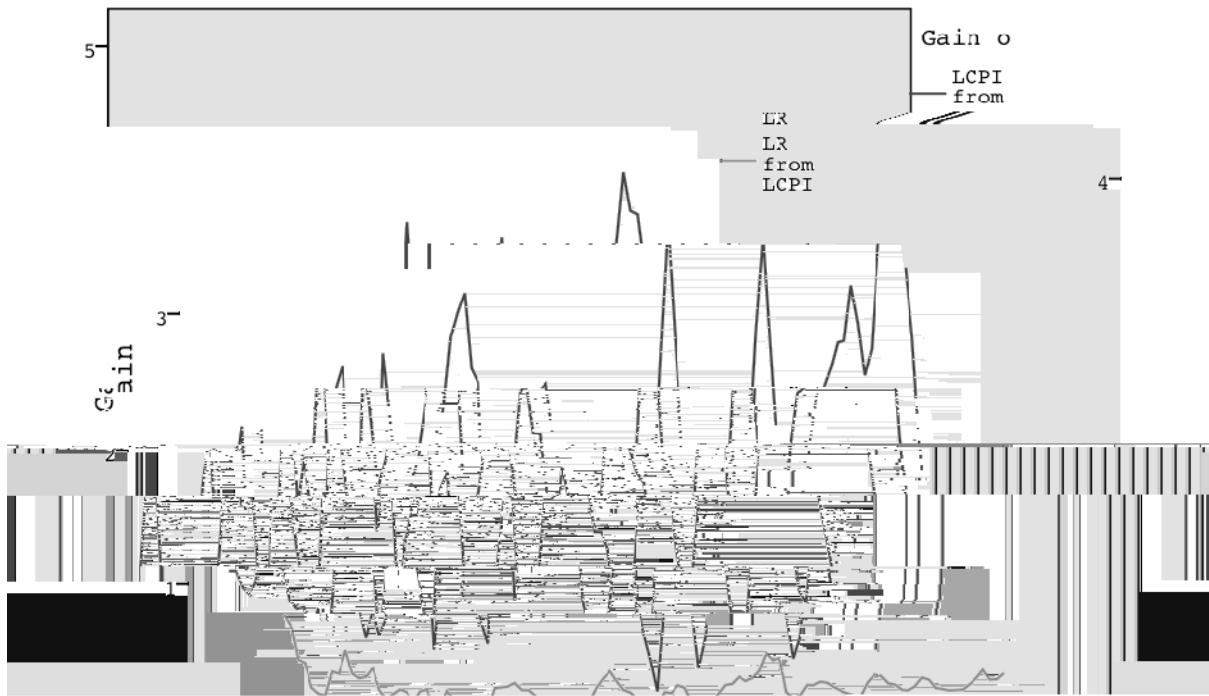


Figure 5

*Squared Coherency of CPI and LR by Frequency*

Source: Authors' calculations

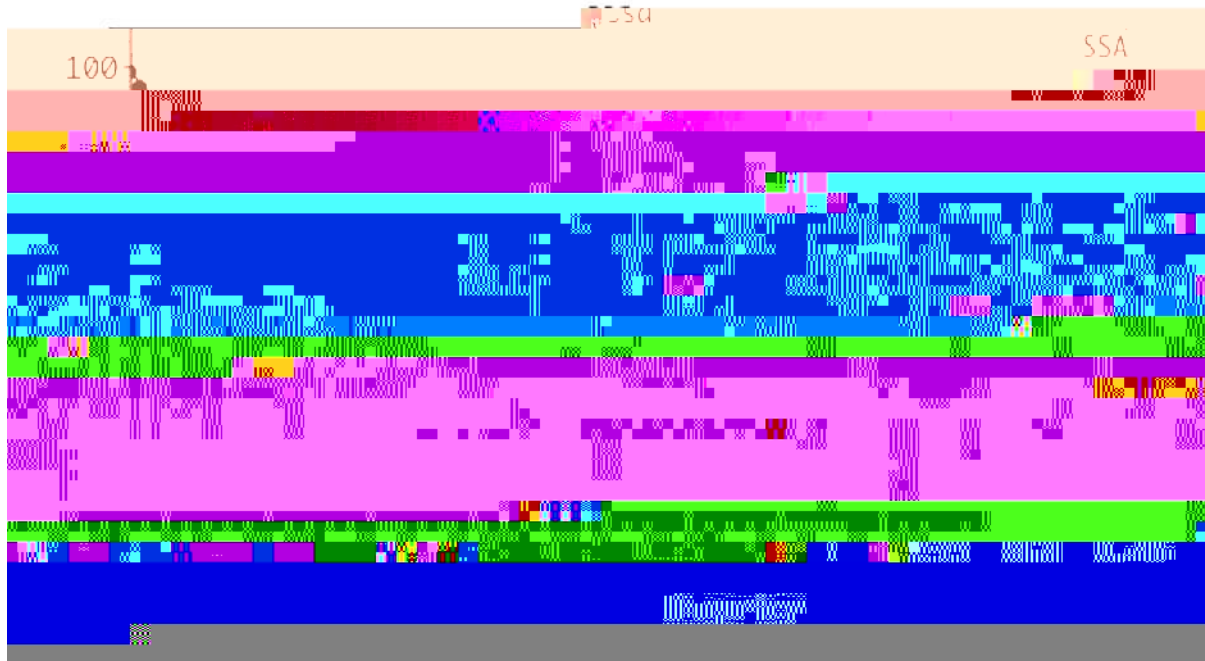
Figure 6

*Gain of CPI and LR*

Source: Authors' calculations

Figure 7

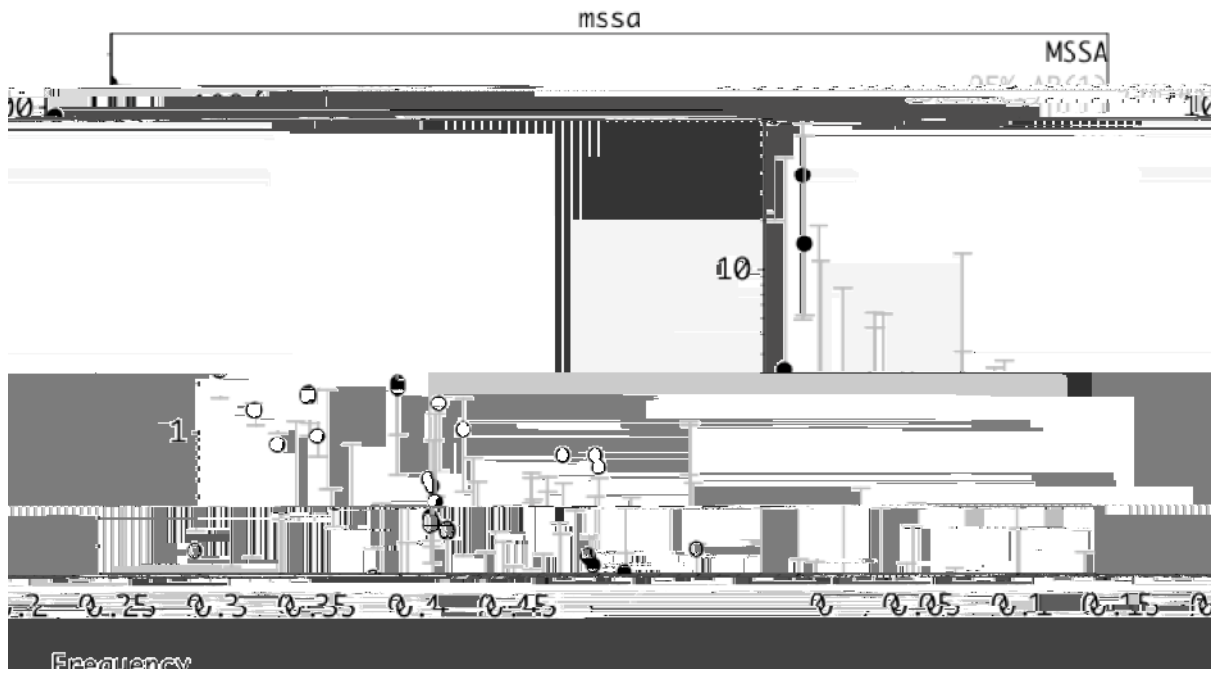
*Singular Spectrum Analysis (Monte Carlo significance test) of Long Term Interest Rates (LR) in Netherland 1800-2012*



Source: Authors' calculations

Figure 8

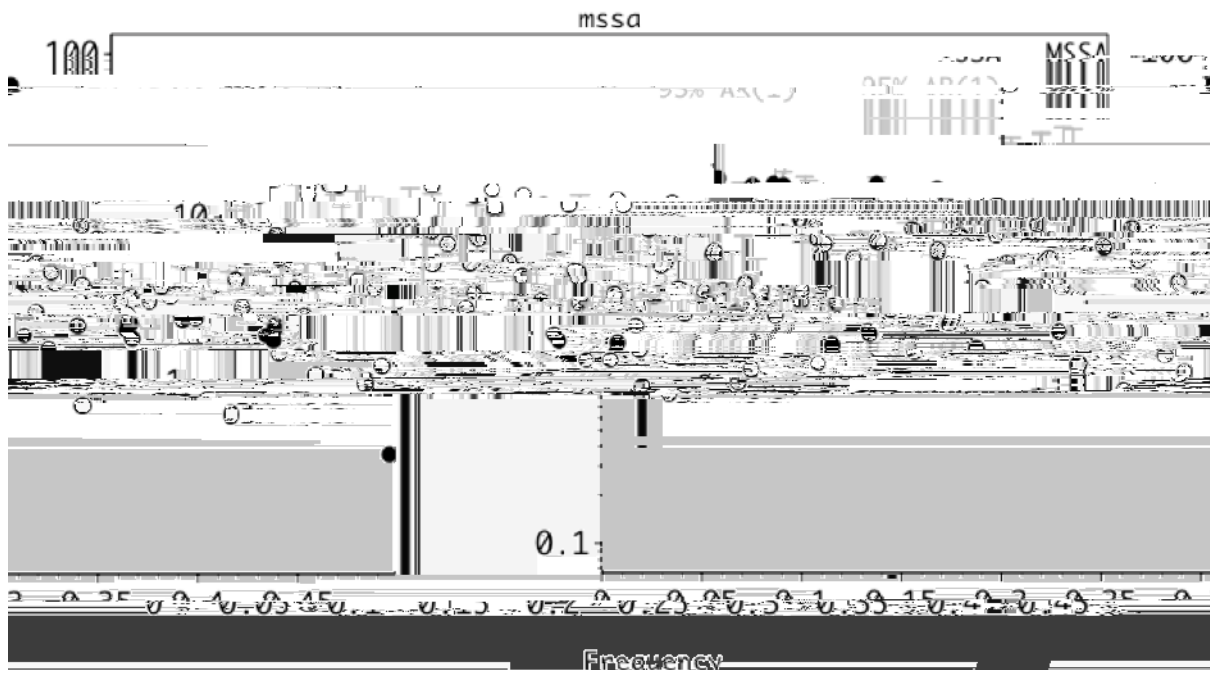
*Multichannel Singular Spectrum Analysis (Monte Carlo significance test) of Long Term Interest Rates (LR) and Prices (LCPI) in Netherland 1800-2012*



Source: Authors' calculations

Figure 9

*Multichannel Singular Spectrum Analysis (Eigenvalues) from a Global M-SSA of all 15 time series with  $m = 50$*



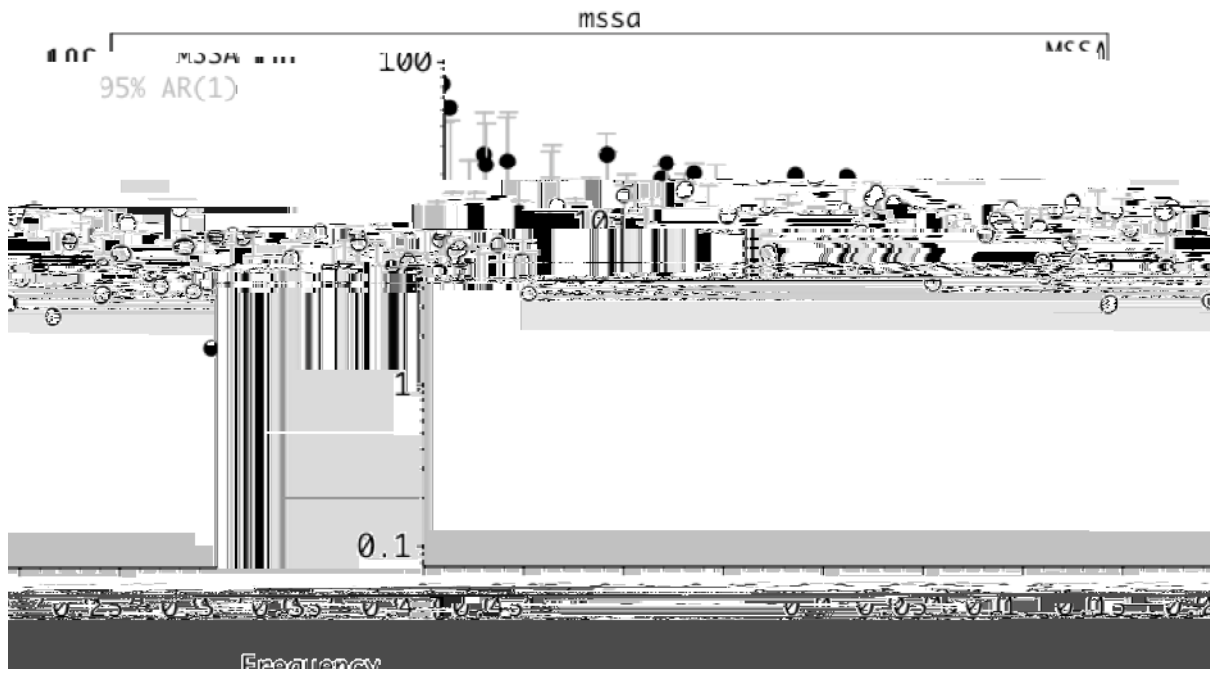
Source: Authors' calculations

Notes: The error bars indicate the 2.5% and 97.5% percentiles of 1000 surrogate time series



Figure 10

*Multichannel Singular Spectrum Analysis (Eigenvalues) from a Global M-SSA of all 15 time series with  $m = 100$*

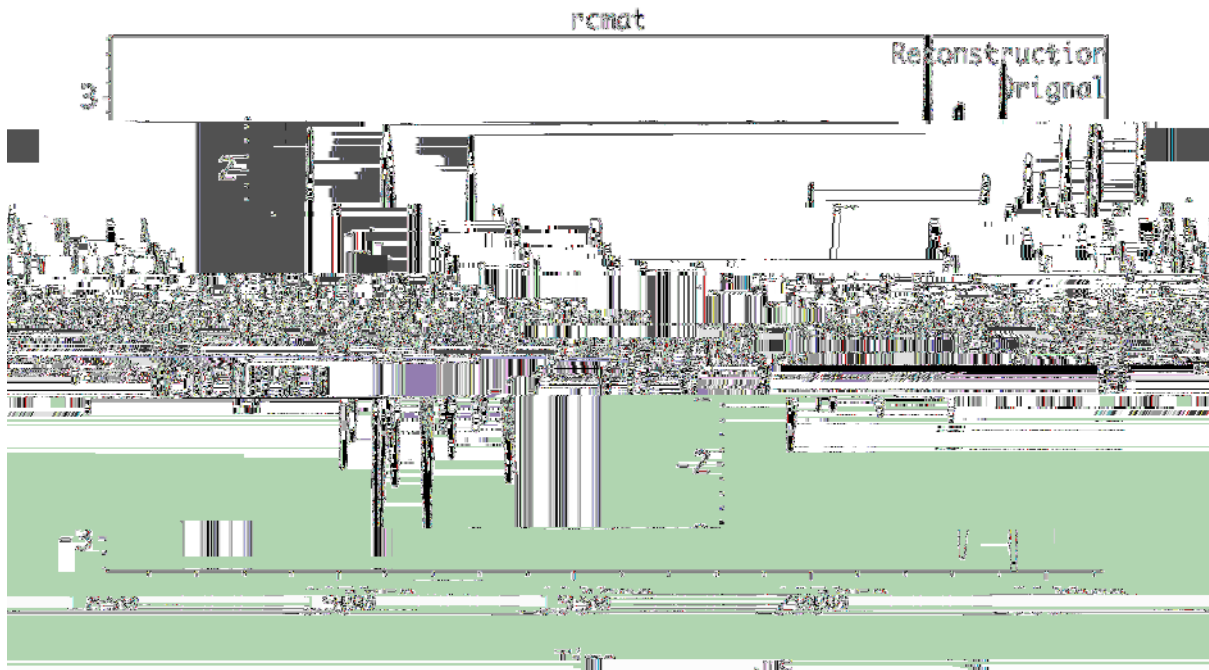


Source: Authors' calculations

Notes: The error bars indicate the 2.5% and 97.5% percentiles of 1000 surrogate time series

Figure 11

*Multichannel Singular Spectrum Analysis Reconstruction for Bond Yields with RC all 15 time series with  $m = 100$*



Source: Authors' calculations