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Audit Probability Versus Effectiveness: The Beckerian Approach Revisited

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The Beckerian approach to tax compliance examines how a tax authority can maximize social welfare by trading-off audit probability against the fine rate on undeclared tax. This paper offers an alternative examination of the privately optimal behavior of a tax authority tasked by government to maximize expected revenue. The tax authority is able to trade-off audit probability against audit effectiveness, but takes the fine rate as fixed in the short run. I find that the tax authority's privately optimal audit strategy does not maximize voluntary compliance, and that voluntary compliance is non-monotonic as a function of the tax authority's budget. Last, the tax authority's privately optimal effective fine rate on undeclared tax does not exceed two at interior optima.

JEL Classification: H26; D81; D63
Keywords: Tax evasion, Tax compliance, Audit probability, Audit effectiveness, Revenue maximization, Probability weighting, Taxpayer

The economics of tax compliance has at its foundations the seminal analy-

tax authority. However, income - a random variable in Reinganum and Wilde (1985) - is, in my model, an exogenous variable, equal across taxpayers. This simplification implies that random auditing is weakly optimal, which moves the focus of the model away from the problem of optimal audit selection towards the problem of how to set a common audit probability, given the reaction function of taxpayers and the trade-off between audit probability and effectiveness. By contrast, when taxpayers differ in income, Reinganum and Wilde (1985) show that there exist audit strategies which condition on taxpayers' reported incomes (such as a cutoff rule) that may dominate a random audit strategy.

Although I shall argue that my approach is consistent with that of Becker, I nevertheless demonstrate that it gives rise to a number of descriptively important differences in prediction. First, the expected-revenue maximizing audit strategy does not maximize voluntary compliance. Instead, the optimal audit probability exceeds that consistent with the maximization of compliance such that, in equilibrium, a marginal increase in the probability of audit reduces declared income.

Second, although the tax authority still has an incentive to raise the fine rate if it is able, Becker's 'hang 'em with probability zero' equilibrium does not emerge. Rather, at all interior solutions of the model, the optimal 'effective' fine rate on undeclared tax does not exceed two. Third, compliance is non-monotonic in the tax authority's budget.

As extensions to the basic model I investigate the implications for my results if taxpayers exhibit probability weighting of the form supposed by prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), and if taxpayers are uncertain as to the true audit probability or effectiveness.

The plan of the paper is as follows: Section 2 motivates the main aspects of cision, and the tax authority's optimal audit strategy. Section 4 analyzes the main results, and Section 5 provides some extensions. Section 6 concludes.

In modern government, responsibility for the collection of taxes is often de-budget constraint." Expected revenue maximization is also assumed as the tax authority's objective function in the literature on optimal audit rules (e.g. Graetz et al., 1986; Reinganum and Wilde, 1985, 1986). Accordingly, in what follows I assume the remit of the tax authority is to maximize expected revenue.

Tax authorities must compete with other government agencies for a budget settlement. Again, this implies that, although the tax authority's budget is endogenous at the level of government, it is largely exogenous to the tax authority itself - at least in the short run. The problem facing tax authorities is therefore to maximize tax revenue for a given budget. In this sense the

My modelling of the ﬁscal environment is based on that of Yitzhaki (1974). In particular, there are ***n*** taxpayers, each with an exogenous taxable income ***y*** (which is known by the taxpayer but not by the tax authority). The

$$\max E[U] = (1 \quad p)\, U[y \quad x] + pU[y \quad x \quad qf \quad (y \quad x)]\,. \qquad (1)$$

For notational convenience I de…ne

$$W \quad y \quad x; \quad W \quad W$$

A4. $\boldsymbol{h}[\boldsymbol{L}]$ is continuous and twice di¤erentiable for all $\boldsymbol{L}$ 0.

A5. $\boldsymbol{h}[0] = 0$ and lim $_{\text{"}\,1}$ $\boldsymbol{h}[\boldsymbol{L}] = 1$.

A6. $\boldsymbol{h}^{0}[\ ] > 0$.

A7. $\boldsymbol{h}^{00}[\ ] < 0$.

Assumption A4 is a standard technical assumption. Assumption A5 is the idea that if the tax authority does not expend any resource on an audit, it will not detect any non-compliance, but a very resource-intensive audit can ultimately detect all non-compliance. Assumption A6 is that audit e¤ectiveness increases as a function of labor. Last, assumption A7 is that audit e¤ectiveness exhibits diminishing returns to labor. Diminishing returns in this context can arise as, unlike many other types of crime, non-compliance takes a great many shapes and forms, each of which di¤ers according to the

$$q = h[L] = h\left[\frac{}{p}\right], \tag{7}$$

where $\quad b/n$ is the per-capita budget of the tax authority. The inverse relationship between p and q makes clear the trade-off in audit strategy between audit probability and effectiveness. Differentiating (7) I have that

$$\frac{\partial q}{\partial p} = \frac{\partial h[\ /p]}{\partial p} = \qquad \frac{q}{p} \quad e \ < 0, \tag{8}$$

where $e\ [L] \quad Lh'[L]/h[L]$ is the elasticity of audit effectiveness with respect to labor and satisfies $e \quad (0;1)$.[2]

I am now able to bring together the budget constraint $q = h[\ /p]$ and the taxpayer behavioral function $x\ [p; q f]$ to define a function $X[p; f]$ $x\ [p; h[\ /p] f]$ that describes the compliance behavior of taxpayers, taking explicit account of the endogeneity of the effective fine rate.

The problem facing the tax authority is to choose the audit probability so as to maximize expected revenue, subject to its budget constraint and its understanding of the behavioral response of taxpayers (as summarized by taxpayers' first order condition). Expected revenue is composed of that generated directly in fines from non-compliance detected at audit (direct effect),

@***E*** [***R***]

empirically, I would expect observed values of to be consistent with an interior solution for compliance. In this sense, while a corner solution for compliance remains a theoretical possibility, from a positive standpoint, analysis pertaining to interior equilibria of the model is of greater signi…cance. This point is of importance in what follows, as the analysis makes strong predictions about behavior in all equilibria with an interior solution for compliance.

The problem in (9) is not a standard concave maximization problem in that the objective function is convex and the constraint function is neither globally concave nor convex (Figure 1). I am nevertheless able to state my …rst Proposition, establishing the existencet

i) _ *the equilibrium satis...es* $p = 1$, $q = h[\ ]$, $x = 0$;

ii) ¯ *the equilibrium satis...es* $ph[\ =p]f = 1$, $x = y$.

In part (***i***) of the Proposition, the tax authority is insu¢ ciently resourced to generate a positive indirect e¤ect, so seeks solely to maximize the direct e¤ect. This is achieved by maximizing the value of $ph[\ =p]$, which implies $p = 1$. By contrast, in part (***ii***), the indirect e¤ect is maximal, and the direct e¤ect is zero.

In this section, I explore the properties of interior solutions of the model in order to contrast the predictions ‡owing from the taxpayer behavioral function $x\ [p; qf]$, which has all the properties of the standard portfolio model, with the equilibrium predictions of the full model, as represented by $X\ [p; f]$.

A well-known prediction of the standard model is that an increase in audit probability increases compliance, i.e. $@x\ [p; qf] = @p > 0$. However, the ceteris paribus condition under which qf is held constant implicitly presupposes an accompanying increase in the tax authority's budget. Under the extension to balanced-budget analysis I obtain the following Proposition:

At all interior equilibria an increase in audit probability decreases compliance: $\frac{[\ ]}{} < 0$.

Proposition 3 follows immediately from the tax authority's ...rst order condition in (10). The ...rst term in (10) is the marginal change in the direct e¤ect from an increase in p, while the second term captures the marginal change in the indirect e¤ect. The former e¤ect is always positive, while the latter takes the sign of $@X\ [p; f] = @p$. For $@X\ [p; f] = @p > 0$ both the indirect and direct effect are increasing in p, so $\partial X[p;f]/\partial p > 0$ is never optimal. By similar reasoning, $\partial X[p;f]/\partial p = 0$ (the compliance maximizing choice of p), is never optimal. Instead, the optimal audit probaility must be such that $\partial X[p;f]/\partial p < 0$. At the optimal audit probability the marginal increase in the direct effect is fully offset by the marginal decrease in the indirect effect, so not only is the indirect effect negative at an interior optimum, it is also strong enough to offset the direct effect.

An implication of Proposition 3 is that audit probability is optimally set *higher* than the compliance maximizing level, and audit effectiveness is set *lower* than the compliance maximizing level. This suggests a tension between the role of the tax authority as a law enforcer (as envisaged by Becker), and as a revenue raiser: to maximize expected revenue the tax authority finds it optimal to tolerate a degree of non-compliance that it could, if it chose, prevent.

The Proposition relies both on the assumptions that the tax authority maximizes expected revenue and that audit effectiveness is endogenous. First, were the tax authority assumed to maximize compliance, then $\partial X[p;f]/\partial p = 0$ would, by assumption, define the optimal choice of p. Second, if audit effectiveness were to be assumed exogenous, which is equivalent to setti6(i)6(t)35(y)-286(m)44(

For $\boldsymbol{f}$ $\underline{\boldsymbol{f}}$, we have $\boldsymbol{p} = 1$ from Proposition 2, in which case to have $\boldsymbol{T}[\boldsymbol{x};\boldsymbol{p}] < 0$ in (3) requires $\boldsymbol{qf} < 1$. For $\boldsymbol{f}$ $\underline{\boldsymbol{f}}$

probability must be increasing as ¯. Similarly for q, I have from (12) that q =¯ = 1=f, but the interior conditions imply q > 1=f, so audit e¤ectiveness must be decreasing as ¯. Formally, a necessary and su¢ cient condition for these two results is that =p is decreasing in (@p=@ > p=) as ¯. The proof proceeds by contradiction to show that if @p=@ = p= as ¯, then the respective …rst order conditions for the taxpayer and the tax authority are not simultaneously satis…ed.

The comparative static results for p and q are proved only local to = ¯, for model complexity frustrated all attempts at a more general result. However, Figure 2 depicts the optimal audit regime for a simulation of the model with logarithmic utility, $U[y] = \ln y$, (which implies constant relative risk aversion) and exponential audit e¤ectiveness, $h[L] = 1$ e ^{2}. For this simple speci…cation of the model, and choosing reasonable values for the …ne and tax rates ($f = 1.5$, $= 0.3$), p and q respond monotonically to over the whole interval [_; ¯].[4] In these cases audit e¤ectiveness is an inferior input in the 'production' of expected revenue.

The …nal result in Proposition 5 is that optimal compliance is non-monotonic in near = ¯ (Figure 3). Although optimal compliance is seen to fall in this region, nevertheless expected revenue continues to increase: the tax authority chooses to allow non-compliance to increase in response to an increase in , even though it could choose to allow it to decrease. Some intuition from the result is seen by
] = 185034] d [(62 (1)]TJ/F14 11.9552 Tf 34.229 0 Td34.2050)]TJ/Fp2 116F22 11.9552

pqf 1, so, from (13), the compliance-independent component accounts for an increasing proportion of total expected revenue. In the limit, the costs of lowering $X[p; f]$ become dominated by the gains from increasing the compliance-independent component of expected revenue.

A prominent feature of descriptive accounts of decision-making under risk is that individuals tend to overweight unlikely outcomes and underweight likely

If taxpayers transform the objective audit probability according to $\mathbf{w}[\mathbf{p}]$ *then for* $\mathbf{p}$ $(0;1)$*:*

***i**) As* $\mathbf{p}$ 0 *it holds that* $\mathbf{p} > \mathbf{p}$;
***ii**) If* $\mathbf{p} = \mathbf{p}$ *then* $\mathbf{p} < \mathbf{p}$;
***iii**) As* $\mathbf{p}$ 1 *it holds that* $\mathbf{p} > \mathbf{p}$.

p

Under p-uncertainty it holds that $p = p$ and $q = q$.

Proposition 7 demonstrates that the analysis of Section 4 is robust to taxpayer uncertainty over p. The result arises as a straightforward consequence of the linearity of taxpayers' expected utility in audit probability. Formally, suppose p is distributed according to P [″], then taxpayers' expected utility is

$$E[U] = U[W$$

Menezes et al. (1980) term downside risk aversion.[5] Together, assumptions A2 and A3 therefore imply that $U'''/U'' > 0$, a property Kimball (1990) terms prudence.

However, in order to sign the fourth derivative of utility, I introduce the

itably explore. For instance, a key assumption one would like to relax is that of homogeneous taxpayers, which in turn might allow for an integration of the present approach with the literature on the design of audit selection rules. The model can also be used to derive policy implications for tax authorities considering changes to their audit portfolio through, for instance, the introduction of 'light-touch' audits - audit types that can be performed quickly and cheaply - as a partial replacement for (longer and more expensive) traditional audit types.

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Existence: I begin by showing that $\lim_{\#0} G[X;p] > 0$. As p 0 I have that $h[\;=p]$ 1 and e 0. Therefore, (11) gives
$\lim_{\#0} @X[p;f]=@p = \lim_{\#0}(\;=D)(U^0[W] + (f \quad 1)U^0[W]) > 0$,
which, in turn, implies that $\lim_{\#0} G[X;p] = n \lim_{\#0} \; W \quad W + \frac{[\;]}{} > 0$. I now show that $G[X;p] < 0$ where $p = (h[\;=p]f \quad 1)=(h[\;=p]f \quad 1 + e\;) < 1$. Setting $G[X;p] = 0$ in (10), and substituting for $\frac{[\;]}{}$ from (11) I obtain:

$$(W \quad W) \quad \frac{(1 \quad p)(1 \quad e\;)U^{00}[W]}{(qf \quad 1)\; e\; (1 \quad p) \quad p(qf \quad 1)\; U^{00}[W]}$$
$$= (1 \quad pqf)\; U^0[W] \quad 1 \quad qf(1 \quad e\;)\; U^0[W] \qquad \text{(A.1)}$$

Suppose, by contradiction, that $e = p(qf \quad 1)=(1 \quad p)$, then substituting in (A.1) obtains $(W \quad W)U^{00}[W] = (qf \quad 1)(U^0[W] \quad U^0[W])$, which is a contradiction since the l.h.s. is negative and the r.h.s. is positive, implying $G[X;p] < 0$. It follows, by continuity, that there exists a p satisfying $p > 0$ and $p < (h[\;=p]f \quad 1)=(h[\;=p]f \quad 1 + e\;)$ such that $G[X;p] = 0$.

Uniqueness: I …rst show that $E[R]$ is a convex function of $(x;p)$: the determinant of the Hessian matrix is $H = (fn\,@(ph[\;=p])=@p)^2 > 0$. The iso-expected revenue curves in Figure 1 are therefore concave to the origin. The constraint $X[p;f]$ is not globally concave because, taking q as constant, compliance is an increasing and convex function of p. Since q is approximately constant close to unity, $X[p;f]$ is increasing and convex for p su¢ ciently close to zero. However, to generate multiple equilibria would require $X[p;f]$ to be downward sloping on the convex interval, and for the convex interval to be sandwiched between two concave intervals, neither of which is the case.

It remains to check whether the constraint and objective functions coincide at more than a single point on the interval where both are concave. To see this is not the case, note that iso-expected revenue intersects the line $x = 0$ for $p = p$, where $p \;\; = 1 = h[\;\; = p\;\;]\, f$. The constraint $X[p; f]$ intersects $x = 0$ for $p = p$ (which may not be unique), where $(1 \quad p\;)\, U^0[y]$ $p\;\; (h[\;\; = p\;\;]\, f \quad 1)\, U^0[y\,(1 \quad h[\;\; = p\;\;]\, f\;\;)] = 0$. Substituting p into the de…nition of p yields $((h[\;\; = p\;\;]\, f \quad 1) = h[\;\; = p\;\;]\, f)\,(U^0[y] \quad U^0[y\,(1 \quad h[\;\; = p\;\;]\, f\;\;)]) <$ 0, from which it follows that that $p \;\; < p$.

Part (i): If $x = 0$ then $E[R] = pqf\;\; y$. Since $@(pq) = @p = q + p\,(@q = @p) =$ $q\,(1 \quad e\;) > 0$ it follows that $@E[R] = @p > 0$, implying a corner solution at $p = 1$.

Part (ii): If $pqf = 1$ is feasible ($\quad ^-$) then there is always a solution to $G[X; p] = 0$ in (10), since it implies that $x = y$, so also $W \;\; = W$.

From (10) it is immediate that $G[X; p] = 0$ implies

$@X[p; f] = @p = \quad (W \quad W\;)\,(1 \quad e\;) = \quad (1 \quad pqf\,) \;\; < 0.$

From (5) an interior equilibrium for compliance must satisfy $qf < p^{\;\;1}$. I now show that all interior equilibria also satisfy the inequality $qf < (1 \quad p)^{\;\;1}$. Suppose, by contradiction, that $qf = (1 \quad p)^{\;\;1}$, so $p = (qf \quad 1) = qf$ and $pqf = qf \quad 1$. Substituting $p = (qf \quad 1) = qf$ in (3) gives $U^0[W\;] \quad (qf \quad 1)^2\, U^0[W\;] =$ 0. Now also suppose $\;\; = \_$ which implies $e \;\; = pqf$. Substituting for e in (A.1) I obtain

$$G[X; p] = 0 \qquad (W \quad W\;)\;\; (1 \quad p)\, U^{00}[W\;] \quad p\,(qf \quad 1)\, U^{00}[W\;]$$
$$= U^0[W\;] \qquad 1 \quad qf\,(1 \quad pqf\,) \;\; U^0[W\;]. \quad \text{(A.2)}$$

Part (

$$\int (\varepsilon f \quad 1) \; e \; (1 \quad p) \quad p(\varepsilon f \quad 1) \; U''[W \; [\varepsilon]] \, dQ[\varepsilon]$$

$$> (qf \quad 1) \; e \; (1 \quad p) \quad p(qf \quad 1) \; U''[W] \, .$$

But then (A.1) and (A.5) cannot hold for $(p \; ; x \;) = (p \; ; x \;)$ as the l.h.s. of (A.5) is smaller than the l.h.s. of (A.1), while the r.h.s. of (A.5) exceeds the r.h.s. of (A.1). Instead, it must hold that $\partial X[p; f]/\partial p > \partial X \; [p; f]/\partial p$. In order to restore (10) it must hold that $p \; < p \;$, which implies $q \; > q \;$ and, as $\partial X[p; f]/\partial p < 0$, $x \; > x \;$.

List of Figures

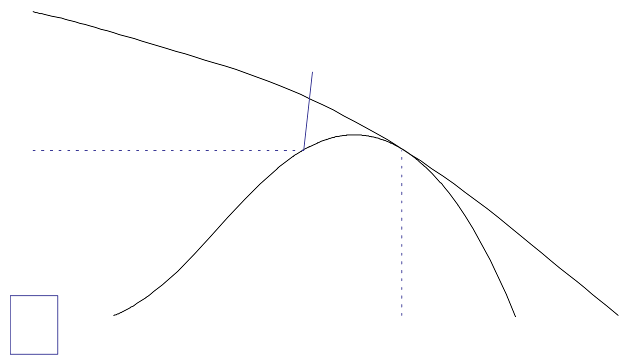

Figure 1: Equilibrium between taxpayers and the tax authority.

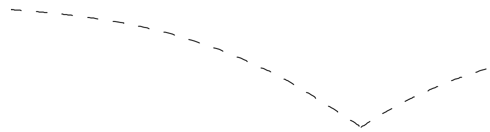

Figure 2: Optimal audit probability and e¤ectiveness (for CRRA utilty and ***h*** [***L***